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Synthetic generation of equilibrium boundary layer turbulence from modeled statistics

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ABSTRACT

An improved synthetic digital filtering method (SDFM) for the generation of equilibrium turbulence is described, with emphasis on the ease of application to complex configurations. This is achieved by facilitating the use of modeled (RANS) statistics for the mean flow and Reynolds stress profiles, instead of simulated (DNS or LES) statistics that are not readily available for three-dimensional flows. The effects on boundary layer relaxation with modeled statistics are first quantified for a well-known canonical situation, specifically, Mach 2 equilibrium turbulent boundary layers over $250 < Re_\theta < 2220$. Various convergence criteria are assessed in terms of the streamwise coordinate (x) normalized by the incoming boundary layer thickness (δ_0). These include skin-friction, which requires $x/\delta_0 \approx 12$ for equilibrium, inner scale mean velocity ($x/\delta_0 \approx 16$), inner scale Reynolds stresses ($x/\delta_0 \approx 18$), as well as outer scale mean velocity, Reynolds stresses, and spatio-temporal correlations ($x/\delta_0 \approx$ 40–50). Development lengths are shown to be inversely related to Reynolds number. In terms of boundary layer relaxation length, initializing the method with RANS (versus simulated) statistics incurs a negligible penalty in inner coordinates and a mild penalty ($x/\delta_0 \approx$ 5–10) in outer coordinates. The effectiveness of this approach is then demonstrated by application to significantly more complex three-dimensional flows, including internal single and multi-stream flows, for which synthetic turbulence generation techniques are necessary, but precursor high-fidelity simulations are not available. Thus, the RANS-initialized SDFM provides a rapidly adaptable method for the synthetic generation of wall-bounded turbulence.

1. Introduction

Scale-resolving simulations of spatially evolving turbulent flows often require the specification of equilibrium inflow turbulence to reduce computational resource requirements. The generation of equilibrium turbulent boundary layers is of particular interest, given the expense of simulating wall-bounded flows with high-fidelity methods such as Direct Numerical Simulations (DNS) or Large-Eddy Simulations (LES). Specifying such a boundary layer, as opposed to simulating the actual process of natural or bypass laminar-to-turbulent transition, is especially appropriate for large Reynolds number. Common techniques that promote development of equilibrium turbulence by introducing fluctuations on the inflow boundary include recycling/rescaling and synthetic generation methods, of which recent developments and variations are discussed by Wu [1], Morgan et al. [2] and Dhamankar et al. [3]. Particularly, the interest in synthetic generation techniques has increased recently, as LES analyses of applied configurations have become more practical [4–7]. All of these methods require some *development length* to allow the turbulence to relax to an equilibrium boundary layer downstream of the inflow plane. Distinctions can be made between the methods [2], classifying each based on the required development length, initial temporal transient (for recycling/rescaling), and *a priori* provision of turbulent statistics (for synthetic generation); the advantages and disadvantages of each method must be considered with respect to the application of interest.

Recycling/rescaling methods are advantageous as they result in the shortest development length and do not require the *a priori* provision of turbulent statistics; however, they have several disadvantages. First, since turbulent statistics are not provided, they

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must be acquired during a potentially long initial transient to ensure that rescaling is applied appropriately; this transient can require 50 flow-through-times across the recycling region [2]. Additionally, improvements to recycling/rescaling methods that reduce spurious low-frequency content related to the recycling frequency rely on the assumption of spanwise homogeneity of the flow. Finally, an equilibrium region is required, over which the similarity profile is valid and in which rescaling can be applied; this condition is not generally applicable for boundary layers with non-constant streamwise pressure gradients or spanwise inhomogeneity, including boundary layers with curvature, converging/diverging nozzles, and other application driven geometries. Therefore, synthetic generation methods are necessary for these flows.

Synthetic generation methods introduce velocity fluctuations, which conform to specified Reynolds stresses, and associated specified mean flow profiles. Two major classes of synthetic methods are the *synthetic digital filtering method* [8–10] (SDFM) and the *synthetic coherent eddy method* [11,12] (SCEM); these differ in the nature by which coherence is introduced to the fluctuating fields. The former strategy introduces spatial/temporal coherence to randomly generated fluctuating fields through a low-pass filtering procedure, requiring only the prescription of local integral length scales, while the latter requires a more detailed Lagrangian description of the anticipated eddy structure of the flow. The development length required for the SDFM applied in this work ($12\delta_0$ for skin-friction and $18\delta_0$ for inner scale velocities and Reynolds stresses, where δ_0 is the incoming boundary layer thickness) is similar to previous reported implementations of the SDFM [10,13,14], as well as the original SCEM [11], which imposes relatively simple Gaussian turbulent spots. Further reduction in the development length may be achieved by advanced SCEM methods, such as that proposed by Pamies et al. [12], which includes four modes with shape functions individually tailored to reproduce eddy behavior in the inner, log, and wake layers of the turbulent boundary layer. Development lengths for these methods can be as small as 5–$6\delta_0$; however, these methods require greater *a priori* knowledge of eddy shape functions, which are anticipated to vary when more complex flows are considered, including corners and those with significant pressure gradients. The SDFM is therefore more suitable for general purposes, due to its robustness and simplicity, in that it only requires the specification of integral length scales, in addition to the first and second-order turbulent moments. Furthermore, the relaxation length of the SDFM is relatively insensitive to the prescribed integral lengths [10,13], which can be taken as constant relative to the outer scale of the flow.

There are three primary objectives to this work. We first describe and validate our implementation of the SDFM, which builds on recent developments in the literature [10,14], with additional improvements to the filtering procedure. For this purpose, we simulate a canonical zero-pressure-gradient, Mach 2, turbulent boundary layer and examine the relaxation to equilibrium of the developing boundary layer with a high level of scrutiny. We focus especially on differences between the outer and inner layers; the fact that the former relaxes more slowly than the latter has been observed in several studies employing both the SDFM [10,13] and SCEM [11,15,16], but has not been scrutinized in much detail.

Next we examine how the choice of turbulent statistics used to initialize the SDFM affects the performance of the method. Although the literature is not always clear on how the required statistics (specifically the mean flow and Reynolds stresses) are selected, it is evident that a variety of approaches have been used. We distinguish between simulated (such as from a precursor LES or DNS) and modeled (such as from a Reynolds-Averaged Navier–Stokes (RANS) calculation) variants – the former yields more accurate statistics, but is also more expensive to obtain. Including both SDFM and SCEM approaches, representative variants of the former have been employed by Klein et al. [8] and Pamies et al. [12], while Larsson et al. [17] and Roidl et al. [16] use the latter. Touber and Sandham [10] as well as Dhamankar et al. [14] use mixed statistics composed of semi-empirical relations for the mean flow combined with DNS or LES Reynolds stresses at a similar Reynolds number.

The primary advantages of using modeled statistics to initialize the procedure are the relative ease and efficiency of RANS in obtaining results at the desired flow condition. Likewise, it is relatively straightforward to obtain these results in more complex configurations such as internal or multi-stream flows. The primary disadvantage is, of course, the use of lower-fidelity statistics. Indeed, for the simple case of an equilibrium turbulent boundary layer, two-equation RANS models can provide the proper mean turbulent boundary layer profiles in the inner layer, since RANS closures are tailored to yield an accurate wall-shear Reynolds stress comparable to that which might be obtained from an LES, but with isotropic (incorrect) normal Reynolds stresses. In particular, the streamwise-normal Reynolds stress is significantly under-predicted, and the mean profiles in the outer layer (wake) are often erroneous. Therefore, when modeled statistics are employed to initialize the SDFM, there is generally a mild penalty associated with the extra relaxation length required for the equilibrium boundary layer to develop. Nonetheless, studies have demonstrated that the SDFM is effective when initialized using RANS statistics, but the additional development length penalty so incurred has not been quantified [17,18]. We remedy this deficiency by initializing the same boundary layer flow with statistics from both approaches, then examine the effect on the development length over which the boundary layer relaxes to its final desired equilibrium state.

As noted, the specified statistics comprise two components: the mean profiles and the Reynolds stresses. There is precedent to using different approaches to specify each of these components independently [10,14], and the question then arises if an optimal method might employ modeled (or semi-empirical) statistics for the mean profiles and simulated statistics for the stresses; this strategy may be viewed as more efficient, since modeled mean flows can be readily obtained, and high-fidelity Reynolds stresses may be available from databases at reasonably similar conditions. We therefore consider the four natural combinations that arise: RANS-RANS, LES-LES, RANS-LES and LES-RANS, where the first designation identifies the method used to obtain the mean flow and the second that for the stresses. For each, we analyze the relaxation of the inner and outer layers to understand the effect of the specified statistics on the relaxation process, including particularly the implications of phase errors in the synthetic fluctuations relative to errors in the specified statistics. We also discuss considerations of whether mixed statistics aid or hinder the development of an equilibrium boundary layer.

Finally, these results are leveraged to generate the required turbulent boundary layers for more complex flows. In particular, we consider internal flows with wall curvature and corners, in which the state of the boundary layer has a significant impact on the subsequent dynamics. These complex configurations necessitate the specification of synthetic turbulence, since significant variation of streamwise pressure gradients and nonhomogeneity obstruct the application of recycling/rescaling methods. We first consider the prediction of mean and fluctuating quantities along the centerline of a Mach 0.9 round jet. This is followed by evaluation of a rectangular multi-stream flow evolving over a deck, which poses a significant challenge, since it incurs the need to specify two independent rectangular inflow streams, each with multiple boundary layers and corners.

We discuss the numerical method in Section 2 and summarize the SDFM implementation in Section 3. The flat plate zero-pressure-gradient boundary layer validation case is described in Section 4.1, followed by SDFM convergence criteria for different