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Simulations of radiation hydrodynamics and radiative magnetohydrodynamics by collocation spectral methods

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A B S T R A C T

During recent years, considerable attention is given to the study of mixed convection and thermal radiation, and the study of radiative magnetohydrodynamics (R-MHD). There are lots of practical applications, such as, nuclear power plants, gas turbines, various propulsion devices for aircraft, missiles, satellites and space vehicles, thermal energy storage, and polymer industry. In the communication of numerical simulation, the solution of coupled heat transfer, fluid flow, thermal radiation and magnetohydrodynamics (MHD) includes solving the Navier-Stokes equations with buoyancy or magnetic force terms, the energy equation with radiative source term, the radiative transfer equation (RTE), and the electrical potential equation. These governing equations are usually partial differential equations except for the RTE which is a integral-differential (first order) equation. It is difficult to use only one numerical method to solve all these equations. Here, we develop the collocation spectral methods (CSM) to solve all above governing equations. If the cases are transient, the projection scheme is adopted for the temporal discretization. For the numerical procedure, the direct solution part can be as much as possible in our projection collocation spectral methods. Due to the numerical simulations of the radiation hydrodynamics (R-HD) and R-MHD with high efficiency and accuracy, lots of complex phenomena can be analyzed successfully and many significant conclusions are obtained.

1. Introduction

Thermal radiation effects on convection and magnetohydrodynamics (MHD) play important role due to the wide applications of radiation hydrodynamics (R-HD) and radiation magnetohydrodynamics (R-MHD), like, the crystal growth in liquid, the cooling of nuclear reactor, the propulsion devices for space vehicles, the solar energy technology, etc. The detail review may refer to [1–4]. The governing equations, which describe the phenomena of R-HD or R-MHD, generally consist of partial different equations of mass, momentum (with source term of buoyancy or magnetic force), energy (with radiative source term), and radiative intensity, and the concentration equation or the Maxwell equation may also be considered if the fluid is compressible or the magneto-fluid is not quasi-static (the induced magnetic field can be neglected). Up to now, there are many numerical methods can be used to simulate the R-HD or R-MHD. While, usually the different methods were used for different phenomena correspondingly even in the same combined system. For instance, Borjini et al. [5,6] used the finite volume method (FVM) to solve the radiative transfer equation (RTE) and the finite difference method (FDM) to solve the vorticity-stream formulation, the concentration equation and the energy equation. Zhang et al. [7] used the FVM to solve the mass, momentum and energy equations, but used the discrete ordinates method (DOM) to solve the RTE. According to our knowledge, Han [8] was the only one using the same numerical method, the SIMPLER algorithm based on the FVM, to solve all the governing equations for thermal radiation effects on MHD flow and heat transfer problem.

It is well known that, the spectral methods can provide exponential convergence and have been widely used in computational fluid dynamics (CFD) [9–11], radiative heat transfer [12–15], and coupled radiation and conduction [16–18]. Recently, the collocation spectral method (CSM) was used to investigate the effects of thermal radiation on MHD boundary layer flows [19–21]. To the best of the authors' knowledge, no one uses the CSM to investigate the R-HD and R-MHD except ourselves [1–4,21].

2. Physical model and governing equations in a cubic enclosure

Figure 1 shows the cubic enclosure, in which the semitransparent electrically conduction fluid is contained, and the medium is viscous and incompressible. The left and right walls are kept

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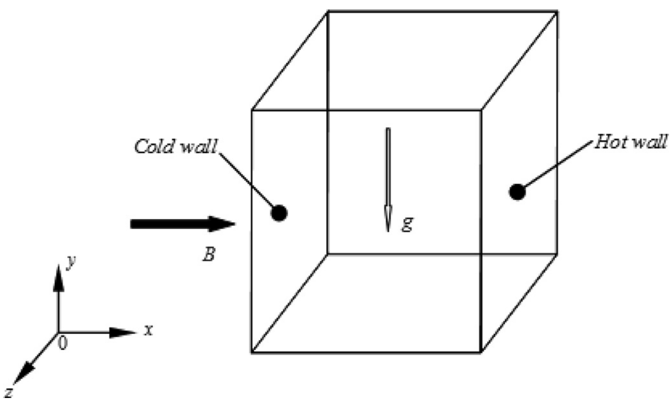

Fig. 1. Cubic enclosure for a R-MHD medium.

at constant lower and higher temperatures, say, T_C and T_H respectively. All other walls are adiabatic. A homogeneous magnetic field is imposed perpendicular to the isothermal walls along the x-direction. All the walls of the cavity are insulating electrically, gray and diffuse.

The fluid properties are taken as constant except for the density, so that the Boussinesq approximation is valid. Neglecting displacement currents, induced magnetic field, viscous dissipations and Joule heating, the non-dimensional governing equations can be written as

$$\nabla\cdot\mathbf{V}=0 \tag{1}$$

$$(\mathbf{V}\cdot\nabla)\mathbf{V}=-\nabla P+PrHa^2(\mathbf{J}\times\mathbf{B})+Pr\nabla^2\mathbf{V}+Pr^2Gr\big[0,\quad\theta,\quad 0\big] \tag{2}$$

$$(\mathbf{V}\cdot\nabla)\theta=\nabla^2\theta-\frac{1}{\delta}\frac{\tau}{Pl}(1-\omega)\left(\Theta^4-\frac{1}{4}G\right) \tag{3}$$

$$\mathbf{J}=-\nabla\tilde{\varphi}+\mathbf{V}\times\mathbf{B} \tag{4}$$

$$\nabla\cdot\mathbf{J}=0 \tag{5}$$

$$\frac{d\tilde{I}}{ds}=(1-\omega)\tau\Theta^4-\tau\tilde{I}+\frac{\omega}{4\pi}\int_{4\pi}\tilde{I}'\Phi d\Omega' \tag{6}$$

Based on Ohm's law, i.e., Eq. (4), the electrical potential Poisson equation can be obtained from Eq. (5), as follows.

$$\nabla^2\tilde{\varphi}=\nabla\cdot(\mathbf{V}\times\mathbf{B}) \tag{7}$$

The following dimensionless variables are defined to get above governing equations.

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ Z=\frac{z}{L},\ U=\frac{u}{u_0},\ V=\frac{v}{u_0},\ W=\frac{w}{u_0} \tag{8}$$

$$s=\frac{s}{L},\ \delta=\frac{T_H-T_C}{T_0},\ u_0=\frac{\alpha}{L},\ P=\frac{p}{\rho u_0^2},\ \theta=\frac{T-T_0}{T_H-T_C} \tag{9}$$

$$T_0=\frac{T_H+T_C}{2},\ \tilde{\varphi}=\frac{\varphi}{Lu_0B_0},\ \tilde{J}=\frac{J}{\sigma u_0B_0},\ \beta=\kappa_a+\kappa_s \tag{10}$$

$$\Theta=\frac{T}{T_0}=\theta\delta+1,\ \tau=\beta L,\ \omega=\frac{\kappa_s}{\beta},\ \tilde{I}=\frac{\pi I}{\bar{\sigma}T_0^4},\ G=\int_{4\pi}\tilde{I}d\Omega \tag{11}$$

$$Pr=\frac{\upsilon}{\alpha},\ Ra=\frac{g\bar{\beta}(T_H-T_C)L^3}{\upsilon\alpha},\ Gr=Ra/Pr,\ Ha=B_0L\sqrt{\frac{\sigma}{\mu}},\ Pl=\frac{\lambda/L}{4\bar{\sigma}T_0^3} \tag{12}$$

where, α, β, τ, ω, φ, I, J are thermal diffusivity, extinction coefficient, optical thickness, scattering albedo, electrical potential, radiative intensity, and electrical current density, and Pr, Ha, Ra, Gr, Pl are the Prandtl number, the Hartmann number, the Rayleigh number, the Grashof number and the Planck number.

The boundary conditions for this system are given as follows for all variables on all six walls.

Velocity: all the walls are non-slip.

Temperature: given temperatures on the left and right walls, zero heat flux on the other walls.

$$\theta=-0.5\quad\text{at}\quad X=0 \tag{13a}$$

$$\theta=0.5\quad\text{at}\quad X=1 \tag{13b}$$

$$-\frac{\partial\theta}{\partial Y}\bigg|_W+\frac{\varepsilon_W}{4\delta Pl}\left(\Theta_W^4-\frac{1}{\pi}\int_{\hat{s}'\cdot\hat{n}_w}\tilde{I}(\vec{r_w},\hat{s}')\left|\hat{s}'\cdot\hat{n}_w\right|d\Omega'\right)=0\quad\text{at}\quad Y=0,1 \tag{13c}$$

$$-\frac{\partial\theta}{\partial Z}\bigg|_W+\frac{\varepsilon_W}{4\delta Pl}\left(\Theta_W^4-\frac{1}{\pi}\int_{\hat{s}'\cdot\hat{n}_w}\tilde{I}(\vec{r_w},\hat{s}')\left|\hat{s}'\cdot\hat{n}_w\right|d\Omega'\right)=0\quad\text{at}\quad Z=0,1 \tag{13d}$$

Electrical current density: zero value on all walls.

Electric potential: zero gradient on all walls.

$$\frac{\partial\tilde{\varphi}}{\partial X}=0\quad\text{at}\quad X=0,1 \tag{14a}$$

$$\frac{\partial\tilde{\varphi}}{\partial Y}=0\quad\text{at}\quad Y=0,1 \tag{14b}$$

$$\frac{\partial\tilde{\varphi}}{\partial Z}=0\quad\text{at}\quad Z=0,1 \tag{14c}$$

The radiative intensity: diffusively emission and reflection.

$$\tilde{I}(\mathbf{r}_W,\Omega)=\varepsilon_W\Theta_W^4+\frac{1-\varepsilon_W}{\pi}\int_{\mathbf{n}_W\cdot\Omega'<0}\tilde{I}\left(\mathbf{r}_W,\Omega'\right)\left|\mathbf{n}_W\cdot\Omega'\right|d\Omega' \tag{15}$$

3. Numerical methods

All above governing equations are steady, while, it is necessary to add the temporal derivatives for all variables for our future unsteady simulations, except the radiative intensity because of the high speed of radiation transfer as light. To treat the coupling of velocity and pressure, the pressure Poisson equation can be obtained by taking divergence of momentum equation with the requirement of incompressibility constraint Eq. (1).

$$\nabla^2P=\nabla\cdot[(\mathbf{V}\cdot\nabla)\mathbf{V}+\mathbf{F}] \tag{16}$$

where, the vector $\mathbf{F}$ contains all the source terms of magnetic and buoyancy forces in Eq. (2).

Boundary condition of pressure can be achieved directly through the arrangement of momentum equation on the boundary as the Neumann case.

For the transient system, the improved projection scheme (IPS), a second-order backward implicit Euler scheme is adopted for the time discretization. The diffusion terms, the pressure gradient, and