Computers and Fluids 162 (2018) 139–151

Contents lists available at ScienceDirect

Computers and Fluids

journal homepage: www.elsevier.com/locate/compfluid

Adjoint-based optimization of a source-term representation of vortex generators

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ARTICLE INFO

Article history:
Received 5 April 2017
Revised 30 October 2017
Accepted 15 December 2017
Available online 16 December 2017

Keywords:
Continuous adjoint method
Source-term model
Vortex generators
Finite-volume method
Computational fluid dynamics

ABSTRACT

An optimization approach is presented that can be used to find the optimal source term distribution in order to represent a high-fidelity vortex-generator (VG) induced flow field on a coarse mesh. The approach employs the continuous adjoint of the problem, from which an exact sensitivity is calculated and used in combination with a trust-region method to find the source term which minimizes the deviation with respect to the reference velocity field. The algorithm is applied to an incompressible flow over a rectangular VG and VG pair on a flat plate and compared to results obtained with the jBAY-model and a body-fitted mesh simulation. The results indicate that a highly accurate flow, yielding only minimal errors with respect to the shape factor, circulation and vortex core, can be obtained on coarse meshes when adding a source term to only a limited number of cells. This approach therefore demonstrates the potential of source-term models to include the effects of VGs in computations of large-scale geometries. It also allows quantification of the achievable accuracy on a particular mesh and the calculation of the source term which is optimal for a specific situation. Furthermore, the optimization approach can be used to diagnose the deficiencies of an existing source-term VG model, in this work the jBAY model.

1. Introduction

Flow-control devices have the capability of increasing the performance of for example wind-turbine blades, engine inlet ducts or airplane wings considerably [1,2]. One of the most commonly used tools in this respect are passive vortex generators (VGs), which consist of small vanes that are mounted on a surface of interest at an angle relative to the incoming flow. VGs introduce streamwise vortical structures that provide mixing in the inner part of the boundary layer with the high-momentum flow of the outer region, thus locally re-energizing the flow. They thereby increase the ability of the flow to overcome adverse pressure gradients and hence reduce its susceptibility to flow separation [3]. By ensuring an attached flow over a larger region of the surface, the addition of VG arrays to e.g. wind-turbine blades increases the maximum lift forces and stall angles, resulting in improved power generation and overall better performance [4–6].

However, reliable predictions of the effects of VGs, and the determination of optimal configurations for specific operating conditions, is not straightforward. These require the ability to accurately model the physics associated with the flow patterns induced by individual VGs and their combined effects when operating in arrays. Computational fluid dynamics (CFD) simulations which resolve the VG geometry could potentially offer the required fidelity, but the computational cost would be excessive, due to the large difference in scale between VGs (with a height typically smaller than the boundary-layer thickness) and the overall structure of interest.

A more feasible approach is to model the effect of the VG on the flow by the addition of a suitable source term to the governing equations. Then only minor or even no mesh adaptations are required with respect to clean-surface simulations, resulting in large savings in computational cost. For example, this can be advantageous for the determination of optimal VG locations. An overview of such source-term VG models is contained in [7]. Common approaches include the addition of a predefined vortex profile based on analytic models [8,9], either as secondary velocities or more recently as additional turbulent stresses [10]. A disadvantage of these methods is that they rely on the inherent assumption that the created vortex can be described by an analytic idealized-vortex model (often the Lamb–Oseen model), usually in combination with empirical relations. Thereby they fail to fully take into account the particular characteristics of the situation of interest, for example the VG geometry and flow conditions.

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https://doi.org/10.1016/j.compfluid.2017.12.009

An alternative consists of including body forces to the governing equations that trigger the formation of a vortex with suitable characteristics [11,12], hence making no prior assumption on the shape and strength of the final vortex. An example is the VG model proposed by Bender et al. [11] (usually referred to as the BAY model), and its improved version the jBAY model [13], which makes use of airfoil theory to estimate the lateral forces generated by the VG. Both the BAY and the jBAY models are commonly used in industry.

Analysis of the BAY and jBAY models by Florentie et al. [14] has shown that this approach performs reasonably well, creating a vortex with characteristics similar to those of a reference body-fitted mesh simulation. However, also some shortcomings are observed. It is found that still a rather refined mesh is required in the vicinity of the VGs, and that the vortex characteristics do not converge to the body-fitted mesh results upon refinement of the mesh. Moreover, analysis of the boundary-layer shape factor revealed deviations that could indicate unreliability with respect to separation prediction. Being constrained by the use of suboptimal meshes, it is unclear whether and to which extent improvement is possible. The question thus arises what is the highest accuracy one can expect to achieve when making use of a source-term model to simulate VG-induced flow effects, on a given mesh? More importantly, which source term would be able to yield this result? In this contribution we present an inverse approach to answer both questions. We perform a goal-oriented optimization, using the adjoint system, to calculate the optimal source-term distribution that recreates the characteristics of a given high-fidelity 3D flow field on a low-resolution mesh.

The formulation of this optimization problem and the derivation of the adjoint system is presented in the next section, followed by details about the numerical implementation. This is followed by an overview of the problems used to test our approach and the derivation of the corresponding boundary conditions. A discussion of the results obtained, with respect to attainable accuracy and source-term patterns, is presented in Sections 5–7.

Note that although the current work is focused on the simulation of VG-induced flow effects, the presented methodology is general. It can in theory also be used to find a suitable source term to replace other small obstacles in CFD simulations.

2. Methodology

2.1. Formulation of the optimization problem

Our objective is to find a source term that accurately reproduces the flow disturbance caused by the presence of a VG. This disturbance consists of a vortex within the boundary layer, propagating downstream and thereby altering the boundary-layer profile. Quantities of interest when analyzing this effect include the (location and value of the peak in the) vorticity field,

$$\omega = \nabla \times \mathbf{u}, \tag{1}$$

and its integral measure accounting for the change in overall flow circulation,

$$\Gamma = \oint_{\mathbf{l}} \mathbf{u} \cdot \mathrm{d}\mathbf{l} = \int_{\mathbf{S}} (\nabla \times \mathbf{u}) \cdot \mathrm{d}\mathbf{S} = \int_{\mathbf{S}} \omega \cdot \mathrm{d}\mathbf{S}. \tag{2}$$

In the above, $\mathbf{u}(x, y, z)$ represents the velocity field in a domain of interest Ω with boundary $\partial\Omega$. If the VG is included for separation control, it is also important to accurately represent its effect on the boundary-layer shape factor, H. The shape factor gives an indication of the boundary layer's ability to withstand adverse pressure gradients (and hence its susceptibility to flow separation). This integral quantity is defined as the ratio between the boundary-layer displacement thickness δ^* and momentum thickness θ, which for incompressible flow is given by

$$H = \frac{\delta^*}{\theta} = \frac{\int_0^\delta \left(1 - \frac{\mathbf{u}}{\mathbf{u}_\infty}\right) \mathrm{d}z}{\int_0^\delta \frac{\mathbf{u}}{\mathbf{u}_\infty}\left(1 - \frac{\mathbf{u}}{\mathbf{u}_\infty}\right) \mathrm{d}z}, \tag{3}$$

where δ represents the boundary-layer thickness and z is the wall-normal direction.

The above measures are a function of the velocity field only. This implies that a source term which provides an optimal match to the velocity field would be sufficient to yield the highest achievable accuracy with respect to the quantities of interest. Goal-oriented optimization using an objective function that minimizes the l^2-norm of the deviation between the velocity field and a high-fidelity reference solution $\tilde{\mathbf{u}}$,

$$J(\mathbf{u}) = \int_\Omega |\mathbf{u} - \tilde{\mathbf{u}}|^2 \,\mathrm{d}\Omega, \tag{4}$$

is therefore expected to be able to yield an optimal source term for representing the effect of a VG on the local flow field. The corresponding source term $\mathbf{f}^*$ can be calculated by solving the constrained optimization problem

$$\mathbf{f}^* = \arg\min_{\mathbf{f}} J(\mathbf{u}) \qquad \text{subject to} \qquad \mathbf{R}(\mathbf{u}, p, \mathbf{f}) = 0 \qquad \text{on} \quad \Omega, \tag{5}$$

with $\mathbf{R}(\mathbf{u}, p, \mathbf{f})$ representing the state equations and boundary conditions to be satisfied by the flow in the domain Ω. Here, $\mathbf{R}$ is defined by the incompressible Reynolds Averaged Navier–Stokes (RANS) equations,

$$\mathbf{R}_\mathbf{u} = (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla p - \nabla \cdot (2\nu D(\mathbf{u})) + \mathbf{f} = 0 \tag{6}$$

$$R_p = \nabla \cdot \mathbf{u} = 0, \tag{7}$$

where $\mathbf{R} = (\mathbf{R}_\mathbf{u}, R_p)^T$, ν denotes the kinematic viscosity (comprising both the molecular and turbulent viscosity) and $D(\mathbf{u}) = \frac{1}{2}(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)$ is the strain tensor. The momentum source $\mathbf{f}$ is defined as

$$\mathbf{f} = C\,\mathbf{f}^0, \tag{8}$$

where $C = \mathrm{diag}(\mathbf{c})$ is a coefficient matrix and $\mathbf{f}_0$ an initial (uniform) forcing which is nonzero in Ω_{VG}, a subdomain enclosing the VG, so that

$$\mathbf{f}^0 = \begin{cases} \mathbf{F}^0/V_{tot} & \text{in } \Omega_{VG} \\ 0 & \text{in } \Omega \setminus \Omega_{VG} \end{cases}. \tag{9}$$

In (9), V_{tot} denotes the volume of Ω_{VG} and $\mathbf{F}^0$ is an initial estimate for the total forcing applied in Ω_{VG}. Optimization of the source-term distribution is achieved by varying the vector of control variables $\mathbf{c}$. This approach is chosen over the direct optimization of $\mathbf{f}$ in order to prevent problems due to poor scaling of the system.

2.2. Derivation of the continuous adjoint system

To solve the constrained optimization problem, Eq. (5) is reformulated as an unconstrained optimization problem using the Lagrange-multiplier method. Using the control variable $\mathbf{c}$ instead of $\mathbf{f}$, we have

$$\mathbf{c}^* = \arg\min_{\mathbf{c}} \mathcal{L}(\mathbf{u}, p, \mathbf{c}, \mathbf{v}, q), \tag{10}$$

with the Lagrange functional

$$\mathcal{L}(\mathbf{u}, p, \mathbf{c}, \mathbf{v}, q) = \int_\Omega |\mathbf{u} - \tilde{\mathbf{u}}|^2 \,\mathrm{d}\Omega + \int_\Omega \mathbf{v} \cdot \mathbf{R}_\mathbf{u}(\mathbf{u}, p, \mathbf{c}) \,\mathrm{d}\Omega + \int_\Omega q R_p(\mathbf{u}) \,\mathrm{d}\Omega, \tag{11}$$

where $\mathbf{v}$ and q are the Lagrange multipliers, often denoted as the adjoint velocity and adjoint pressure respectively. By rewriting the