Computers and Fluids 160 (2018) 14–25

Contents lists available at ScienceDirect

Computers and Fluids

journal homepage: www.elsevier.com/locate/compfluid

A stability analysis of the compressible boundary layer flow over indented surfaces

Jesús Garicano-Mena*, Esteban Ferrer, Silvia Sanvido, Eusebio Valero

ETSIAE-UPM (School of Aeronautics), Plaza Cardenal Cisneros 3, Madrid E-28040, Spain

ARTICLE INFO

Article history:
Received 16 January 2017
Revised 20 September 2017
Accepted 9 October 2017
Available online 23 October 2017

MSC:
76N20
76E19
76E09

Keywords:
Stability analysis
Compressible boundary layer flow
Flat plate
Indentation
High-order numerical method

ABSTRACT

This contribution presents a stability analysis for compressible boundary layer flows over indented surfaces. Specifically, the effects of increasing depth D/δ^* and Ma_∞ number on perturbation time-decay rates and spatial amplification factors are quantified and compared with those of an unindented configuration.

The indented surfaces represent aeronautical lifting surfaces endowed with the smooth gap resulting when a filler material applied at the junction of leading-edge and wing-box components retracts upon its curing process. Since the configuration considered is such that the parallel/weakly-parallel assumptions are necessarily compromised, a global temporal stability analysis is considered in this study. Our analysis does not require a parallel flow constrain, and hence it is believed to be valid when two dimensional effects are relevant.

We find that small surface modifications enhance certain flow instabilities. An increase in Ma_∞ enhances further this behaviour: for the $D/\delta^* = 1.5$, $Ma_\infty = 0.5$ case, amplification factors at a given location can be up to 20 times larger than those corresponding to the unindented case.

1. Introduction

The aeronautics industry has shown an increased interest on natural laminar flow (*NLF*) wings. These wings are carefully designed to maintain the flow under laminar conditions over a relatively large extent of the wing area. The advantage is a lower skin friction and the consequent reduction in fuel consumption; this is achieved at the price of more stringent manufacturing tolerances.

From the manufacturing viewpoint, wings are assembled by joining several components, e.g. the main central wing box and the leading and trailing edges. The fitting between these elements is never perfectly tight: small grooves are always left at the wing-box/leading edge and trailing edge junctions. *Filler* materials, of resinous nature, are applied at these locations to alleviate the misfitting problem. However, since filler materials retract during its curing process, a small, possibly smooth indentation remains. The question arises then, whether this smaller but somewhat unavoidable groove in the wing box/leading edge junction can enhance the growth of boundary layer instabilities. In such case, a significant forward movement of transition location would spoil the effort invested in the design of the natural laminar flow region.

It is well-known that in real swept wings, transition to turbulence is mainly driven by cross-flow and Tollmien–Schlichting instability mechanisms [1]. However, and contrarily to more established wing concepts, *NLF* wings operate at comparatively lower sweep angles [2]; this in turn translates in an increased relevance of Tollmien–Schlichting over cross-flow dominated transition mechanisms.

Spatial growth of Tollmien–Schlichting (or TS) structures is one of the avenues explaining laminar to turbulent transition. Through this mechanism, i.e. the natural transition scenario, the TS waves grow exponentially over a finite length, to then saturate and interact in a non-linear fashion, leading eventually to transition to turbulence. Alternatively, non-linear interactions may appear without a definite preliminary exponential growth phase [3] (hence the term *bypass* transition). These alternative mechanisms are not covered in this work.

In the aircraft industry, it is common practice to employ semi-empirical, but extensively validated methods, to predict natural transition location for flows over wings and fuselages at flight conditions. The most common tools either perform a local stability analysis [1,4] or solve the Parabolised Stability Equations [or *PSE* , 5] upon a base state obtained numerically; application of the e^N criteria [6,7] allows then to predict approximate transition locations for natural scenarios, as long as the parallel or weakly parallel assumptions are fulfilled.

* Corresponding author.
E-mail address: jesus.garicano.mena@upm.es (J. Garicano-Mena).

https://doi.org/10.1016/j.compfluid.2017.10.011
0045-7930/

Whenever small surface imperfections are present, the preliminary exponential growth phase (natural transition) may be compromised, resulting possibly into a different transition location. In this case, the parallel hypothesis is not valid anymore, and classical methods may have difficulties in predicting the modified transition location [2]. Alternative methods are needed to handle these situations.

Zahn and Rist report in [8] a detailed analysis of the effect of deep gaps in laminar-to-turbulent transition for a $Ma_\infty = 0.6$ flow, employing direct numerical simulation. They succeed in identifying an acoustic feedback mechanism between standing waves at the gap and the boundary layer, and derived a model that successfully accounts for amplification factor modifications. They also investigate the transition delay effect induced by a deep cavity placed before a forward-facing step.

An alternative approach, based in the definition of a Local Scattering Problem, has been proposed recently in [9–11]. This method leads to an eigenvalue problem whose solution bridges the spatial behaviour much before and after the scatter location (indentation, bump, different materials junction, ...)

In this context, we propose to quantify the effect of small indentations by using global stability analysis techniques [12], since these do not rely on the parallel or weakly-parallel flow assumptions. Indeed, many contributions describe the application of global techniques – both in its modal and non-modal variants – to study laminar separation bubbles on flat plate (*FP*) configurations, be they generated by a convex bump [13–15], by a concave indentation [16] or by an adverse pressure gradient [17]. Alternatively, direct numerical simulation followed by solution of the linearised Navier–Stokes equations may be employed: e.g. [18] investigates the effect of very small-scale, localised bumps and indentations on the Tollmien–Schlichting waves appearing on a *FP* configuration.

In this work, we aim at studying how the presence of an indentation modifies the stability characteristics of a canonical zero pressure gradient boundary layer (or *BL*) over a flat plate. We specifically seek to quantify the effects of increasing indentation depth and flow compressibility (i.e. Mach number) on the linear stability (i.e. the spectrum and amplification factors) by means of global stability tools. In line with most of the studies mentioned above, the flow is considered bidimensional.

The rest of the document is structured as follows: next section describes the flow configurations considered and presents the tool chain employed in our study. Section 3 gathers the results and discussions. Finally, Section 4 summarises our conclusions.

2. Flow configuration and numerical methods

We study a zero pressure gradient boundary layer flow over a flat plate geometry that includes a smooth groove or indentation. The indentation, of infinite spanwise extent, sits at a certain distance downstream of its leading edge (see Section 2.1 for the problem description). We proceed – as in a classical stability analysis – by obtaining first a steady (numerical) solution to the flow governing equations: the *base* flow (*cf.* Sections 2.2 and 2.3); a linear perturbation of this *basic* flow solution and its subsequent expansion in terms of Fourier modes allows then to assemble a discrete eigenvalue problem, (or *EVP*, *cf.* Section 2.4). The spectral information (eigenvalues and eigenfunctions) retrieved is analysed along two dimensions: on the one hand, the eigenvalue locations in the complex plane; on the other hand, the spatial evolution of individual components along the streamwise direction, as given by their amplification factors (Section 2.5).

Table 1
Configurations considered.

Re_{δ^*}	Ma_∞	L_x/δ^*	L_z/δ^*	x_c/δ^*	L/δ^*	D/δ^*	L/D
610	0.1 and 0.5	400	40	100	50	0, 1, 1.5	∞, 50, 33.3

2.1. Problem description

We consider compressible boundary layer flows with $Re_{\delta^*(x)} \in [610, 1050]$ at upstream Mach numbers $Ma_\infty = 0.1$ and 0.5. The incompressible boundary layer flow over a flat plate configuration is, in the range of $Re_{\delta^*(x)}$ considered, convectively unstable [4], and has been addressed in [19,20].

The Reynolds number is based on a displacement thickness $\delta^*(x)$:

$$Re_{\delta^*} = \frac{\rho_\infty U_\infty \delta^*(x)}{\mu_\infty}, \tag{1}$$

where ρ_∞, U_∞ and μ_∞ are the density, speed and dynamic viscosity upstream.

Fig. 1 shows the computational domain studied: it is rectangular in shape, of length L_x and height L_z, and the air flows from left to right. The leading edge of the flat plate is not simulated, instead a solution to the compressible boundary layer equations at the corresponding Ma_∞ is imposed at the leftmost edge of the computational domain, *i.e.* the inlet. The choice of the domain extent is partly guided by previous results on incompressible *BL* flows at the same Re_{δ^*} [19,20]. Specifically, L_z/δ^* needs to be chosen large enough so neither the *BL* growth nor the global eigenfunctions are artificially constrained, see [12]; this consideration becomes more and more restrictive as both Ma_∞ and D/δ^* increase.

The isolated indentation, when present, is located at a distance x_c from the left edge and is characterised by its breadth L and depth D. The notch considered presents a smooth, Gaussian-like profile given as $z = -D \exp\left(\frac{x-x_c}{L/2}\right)^2$.

All the geometrical parameters defining the problem are nondimensionalised with the mass displacement thickness δ^* at the leftmost edge of the domain. In this study we fix the groove extent L/δ^* and location x_c/δ^* and vary the groove depth D/δ^* and the upstream Ma_∞ number [21]; Table 1 summarises the different configurations considered. Notice that the range for the ratio L/D here included is essentially different from previous studies on rectangular low aspect ratio cavities – $L/D \in (1/5, 4)$ – at high Reynolds and Mach numbers described e.g. in [8,22,23].

2.2. Governing equations

We are interested in flows governed by the compressible Navier–Stokes equations, that once expressed in terms of nondimensional, conserved variables $\mathbf{U} = \left[\rho, \rho\vec{v}^t, \rho E\right]^t$ (mass, momentum and total energy per unit volume), can be written in compact vector form as:

$$\frac{\partial \mathbf{U}}{\partial t} + \mathcal{N}(\mathbf{U}) = \vec{0} \tag{2}$$

where $\mathcal{N}$ is the divergence of the flux tensor:

$$\mathcal{N} = \nabla \cdot \bar{\bar{F}}(\mathbf{U}), \tag{3}$$

and $\bar{\bar{F}}$ gathers convective and diffusive effects:

$$\bar{\bar{F}}(\mathbf{U}) = \begin{pmatrix} \rho\vec{v} \\ \rho\vec{v}\cdot\vec{v}^t + p\bar{\bar{I}} - \bar{\bar{T}} \\ \rho H\vec{v} - \bar{\bar{T}}\cdot\vec{v} + \vec{q} \end{pmatrix}. \tag{4}$$

In Eq. (2) above, p and ρH stand for thermodynamic pressure and total enthalpy per unit volume, respectively; $\bar{\bar{I}}$ is the identity matrix.