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Modified multi-dimensional limiting process with enhanced shock stability on unstructured grids

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ABSTRACT

The basic concept of multi-dimensional limiting process (MLP) on unstructured grids is inherited and modified in this work for improving shock stability and reducing numerical dissipation in smooth flows. A relaxed version of MLP condition, simply named as weak-MLP, is proposed for reducing dissipation. Moreover, a stricter condition, that is the strict-MLP condition, is proposed to enhance the numerical stability. The maximum and minimum principles are satisfied by both the strict- and weak-MLP conditions. A differentiable pressure weight function is applied to combine two novel conditions, and thus the modified limiter is named as MLP-pw (pressure-weighted) limiter. A series of numerical test cases show that MLP-pw limiter has improved stability and convergence, especially in hypersonic simulations. Moreover, the limiter also shows low numerical dissipation in simulating flow fields without shock-waves.

1. Introduction

Unstructured grids are commonly used for the spatial discretization of current industrial computational fluid dynamics codes that simulate aerodynamics or gas dynamics phenomena. The advantages of using unstructured grid include the conveniences in automatic grid generation [1–3], grid adaptation [4–6] and moving mesh techniques [7,8], for complex geometries and flow phenomena. However, the accuracy and stability of unstructured schemes are usually challenged by the irregularity of grid connectivity and deterioration of grid quality [9,10], which are inevitable in the automatic discretization for complicated geometries. Especially, simulating transonic and supersonic flows requires accurate approximation of nonlinear multi-dimensional physical phenomena, such as shock wave, shock waves interaction, and shock-vortex interaction, and thus excellent accuracy and stability are indispensable.

As a key factor that affects spatial accuracy and stability, slope limiters, or for short, limiters, have been investigated for decades. As well known, second-order or higher than second-order schemes suffer from numerical oscillations across discontinuities, a typical one of which is shock wave [11,12]. Therefore, limiters are used to suppress these oscillations while keeping second-order reconstruction in the smooth region of flow field. On structured grids, the finite difference method (FDM) and finite volume method (FVM) have been applied along with mature limiting method based on solid theories. The typical strategy is MUSCL (Monotonic Upstream-Centered Scheme for Conservation Laws) scheme [13] with limiters that subject to TVD (Total variation diminishing) condition [14–16]. However, these structured schemes can not be extended onto unstructured grids directly due to various reasons. Firstly, the schemes for structured grids are usually developed based on one-dimensional analysis and extended to multi-dimensional structured grids by dimensional-splitting, which is infeasible for unstructured grids. Secondly, one-dimensional principles, for instance, the TVD condition, are not necessary feasible on multi-dimensional unstructured grids. An example of Jameson had shown that flow field on which the total variation is smaller could be more oscillatory than flow field on which the total variation is larger [17]. Furthermore, a scheme applying TVD condition will cause accuracy deterioration at extrema even in smooth regions, and thus the TVB [18] and ENO [19] schemes were developed.

By extending Spekreijse's monotone condition [20], Barth and Jespersen designed a limiter on unstructured grids [21], which modifies the piecewise linear distribution at each control volume. Barth-Jespersen limiter removes local extrema and insures stability. However, this limiter shows similar effects as that of TVD condition, which reduces accuracy at smooth extrema. Furthermore, the limiting function of Barth and Jespersen is non-differentiable, and thus the convergence is less satisfactory. Therefore, an improvement was introduced by Venkatakrishnan [22], who used a differentiable function similar to that of van Albada limiter

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[23] which is designed for structured grids. Venkatakrishnan limiter archives better convergence compared with Barth-Jespersen limiter. Whereas, Venkatakrishnan limiter is not strictly monotone, and thus it might produce oscillations across shock wave. Generally speaking, Barth-Jespersen limiter and Venkatakrishnan limiter have been successfully applied on unstructured grids since their inventions.

Many researches have been focusing on the improvement of limiters. In order to reduce the dissipations of two aforementioned unstructured limiters, a strategy was introduced, which is turning off limiter in subsonic region. Nejat and Ollivier-Gooch introduced hyperbolic tangent function in their application of Venkatakrishnan limiter, by which the limiter only activates in limited region [24]. Michalak and Ollivier-Gooch further improved this method [25]. Thereafter, Kitamura and Shima introduced the concept of second limiter, which also uses a hyperbolic tangent function to turn off limiter in stagnation or subsonic zone, but removes predefined parameters [26]. It was proved by numerical results that second limiters can reduce dissipations effectively.

A relatively new method on unstructured grids is MLP (Multi-dimensional Limiting Process) limiter, which was first introduced on structured grids [27,28]. By using the MLP condition which satisfies maximum/minimum principles, MLP limiter properly introduces multidimensional information. Therefore, the method has been showing better accuracy, robustness and convergence in various circumstances. Park, et al. designed unstructured MLP limter [29]. Thereby, Park and Kim [30] had constructed three-dimensional unstructured MLP limiter and proved that the limiter obeys LED (Local Extremum Diminishing) condition [17]. Gerlinger designed a low dissipation MLP limiter, MLP^{ld}, on structured grids, and simulated combustion problem [31]. Do, et al. defined a low dissipation MLP limiter for central-upwind schemes [32]. Kang, et al. [33] reduced dissipation by only turning on the MLP limiter in the vicinity of shock waves/nonlinear discontinuities. MLP limiter had also been developed for higher order unstructured numerical schemes, including Discontinuous Galerkin method [34] and Flux-Reconstruction or Correction Procedure via Reconstruction [35–37]. Li, et al. developed a multi-dimensional limiter, WBAP, which modifies the gradients by a component by component approach [38,39]. This method is not rotationally invariant but shows good accuracy, robustness and convergence in numerical tests.

In spite of the successful applications, there is still room for MLP limiter to improve the stability and convergence, especially for hypersonic flow simulations. Therefore, the presented research is focusing on this topic. This paper is organized as follows. The finite volume method and spatial reconstruction are briefly described in Section 2. Then, the Barth-Jespersen limiter, Venkatakrishnan limiter and MLP limiter are briefly introduced in Section 3, where the differences are emphasised. In Section 4, the presented modifications on MLP limiter are formulated. A series of numerical test cases along with corresponding discussions are given in Section 5. Finally, Section 6 concludes the whole work.

2. Finite volume method and second-order reconstruction

The discretization for the compressible Navier–Stokes equations is introduced as follows. The integral form of the equations is

$$\int_{\Omega} \frac{\partial \mathbf{Q}}{\partial t} d\Omega + \int_{\partial\Omega} [\mathbf{F}_c(\mathbf{Q}) - \mathbf{F}_v(\mathbf{Q})] \cdot \mathbf{n} dS = \mathbf{0}, \tag{1}$$

where $\mathbf{Q}$ are the conservative variables in the flow field, $\mathbf{F}_c(\mathbf{Q})$ is convective flux, and $\mathbf{F}_v(\mathbf{Q})$ is viscous flux, which could be solved by using a central scheme for unstructured grids [40]. In this paper, solutions of the convective flux are emphatically investigated. Therefore, in the following discussions the $\mathbf{F}_v(\mathbf{Q})$ term is neglected, and thus the equations are simplified as Euler equations. $\mathbf{Q}$ and $\mathbf{F}_c(\mathbf{Q})$ are given as

$$\mathbf{Q} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{bmatrix}, \mathbf{F}_c(\mathbf{Q}) = \begin{bmatrix} \rho V_n \\ \rho u V_n + p n_x \\ \rho v V_n + p n_y \\ \rho H V_n \end{bmatrix}, \tag{2}$$

where $V_n = \mathbf{V} \cdot \mathbf{n} = (u n_x + v n_y)$. E is the total energy, H is the enthalpy, given as

$$E = \frac{1}{\gamma - 1} \frac{p}{\rho} + \frac{1}{2}(u^2 + v^2), \tag{3}$$

$$H = E + \frac{p}{\rho}, \tag{4}$$

where γ is the ratio of specific heat. For air at moderate pressures and temperatures one may use $\gamma = 1.4$.

The governing equations are discretized by using cell-centered finite volume formulation which is applied to a polygon computational cell i sharing a interface k with a neighbouring cell j. Therefore, the spatial discretization at cell i for the Euler equations can be expressed as

$$\frac{\partial}{\partial t}(\mathbf{Q}\Omega)_i = -\left(\sum_{k=1}^{N_f} \mathbf{F}_{c,k} \cdot \mathbf{n}_k S_k \right)_i, \tag{5}$$

where $S_k = |\partial\Omega_k|$ is the interface area, $\mathbf{n}_k$ is the unit norm vector outward from the interface, N_f is the interface number of cell i. Although the exact convective flux function $\mathbf{F}_{c,k}$ is nonlinear, it is usually solved by a linearized numerical flux instead of the exact formula [41]. Furthermore, the numerical flux function could be simplified as an one-dimensional scheme that calculates in the direction of vector $\mathbf{n}_k$. In fact, upwind schemes, such as FDS (Flux Difference Splitting) scheme or FVS (Flux Vector Splitting) scheme, are mostly designed based on one-dimensional hypothesis. FDS schemes or FVS schemes could be defined as a function of conservative variables $\mathbf{Q}$, and thus the flux is given as

$$\mathbf{F}_{c,k} = \mathbf{F}_{\mathrm{FDS/FVS}}(\mathbf{Q}_k^+, \mathbf{Q}_k^-, \mathbf{n}_k), \tag{6}$$

where the superscript $(\cdot)^{\pm}$ denote the left and right values of interface k respectively. In the following paragraphs, the subscripts c and k are neglected for simplicity.

The cell interface values are extrapolated from the cell centre values by using gradient ∇q:

$$\begin{aligned} q_k^+ &= q_i + \phi_i \nabla q_i \cdot \Delta\mathbf{r}_{ik}, \\ q_k^- &= q_j + \phi_j \nabla q_j \cdot \Delta\mathbf{r}_{jk}, \end{aligned} \tag{7}$$

where $\Delta(\cdot)_{ik} = (\cdot)_k - (\cdot)_i$ and q could be any of the conservative variables. ∇q is calculated by nodal averaging procedure [42] and Gauss-Green scheme [21], and the slope limiter value ϕ is employed to suppress oscillations at captured discontinuities. In the following sections, the calculation of ϕ will be investigated. Reconstruction becomes conservative if the integration of q over a cell equals to the cell-averaged value, i.e.

$$\bar{q} = \frac{1}{|\Omega|} \int_{\Omega} q d\Omega. \tag{8}$$

The time derivative in Eq. (5) could be solved by explicit and implicit schemes. Due to the limited topic of the presented article, temporal solutions will not be further discussed.

3. Limiters

In order to give the background information of designing the improved limiter, the success classical limiters are briefly introduced in this section, with emphasising important features. For the detailed information readers may examine the original articles introducing the limiters.