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An implicit simplified sphere function-based gas kinetic scheme for simulation of 3D incompressible isothermal flows

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ABSTRACT

In this work, an implicit simplified sphere function-based gas kinetic scheme (SGKS) is presented for simulation of 3D incompressible isothermal flows. At first, the numerical fluxes of governing equations are reconstructed by the local solution of Boltzmann equation with sphere function distribution. Due to incompressible limit, the sphere at cell interface can be approximately considered to be symmetric as shown in the work. Besides that, the energy equation is usually not needed for simulation of incompressible isothermal flows. With all these simplifications, the formulations of the simplified SGKS can be expressed concisely and explicitly. Secondly, the commonly-used implicit Lower-Upper Symmetric Gauss-Seidel (LU-SGS) method is adopted to further improve the computational efficiency and numerical stability of present scheme. In LU-SGS method, only a forward and a backward sweep are needed for marching the conservative variables in time. As a result, the simplified SGKS with the LU-SGS method can be implemented easily. Numerical experiments, including the 3D lid-driven cavity flow and flow over a backward-facing step, showed that the incompressible isothermal flows can be well simulated by the developed scheme and its computational efficiency is significantly higher than that of the original SGKS and the lattice Boltzmann flux solver (LBFS). In addition, it was found that the present scheme with the LU-SGS method is more efficient than that with the explicit Euler method, and the speedup ratio is about 2 to 5.

1. Introduction

In the last few decades, the gas kinetic scheme (GKS) has been developed rapidly and applied in a wide range of fluid flow problems, such as in compressible flows [1–3], chemical reaction flows [4], magneto hydrodynamics [5], rarefied flows [6,7], turbulence flows [8,9], etc. In GKS, the finite volume method (FVM) or the finite difference method (FDM) is usually applied to discretize the macroscopic governing equations and the local solution of Boltzmann equation is utilized to compute the numerical fluxes of conservative variables at the cell interface. Since the numerical fluxes are reconstructed from the local solution of physical equation instead of numerical approximation, the GKS is robust, positively-preserving and satisfies the entropy condition spontaneously [10].

The conventional GKS usually computes the numerical fluxes at the cell interface by the local integral solution of Boltzmann equation with BGK collision model and Maxwellian distribution function [1,2,11–15]. For simulation of compressible flows, the local integral solution of Boltzmann equation is quite complicated due to the discontinuity of conservative variables and their derivatives at the cell interface [11–12]. By assuming that the flow variables and their derivatives at the cell interface are changed smoothly, the conventional GKS has been simplified by Su et al. [16] for simulation of incompressible isothermal flows and by Xu and Lui [17] for solving incompressible thermal flows. These methods can be viewed as the limiting case of conventional GKS with the reduced local integral solution of Boltzmann equation at the cell interface. Subsequently, Xu and He [18] and Guo et al. [19] discarded the energy equation and applied the isothermal distribution function to further simplify the conventional GKS for simulation of incompressible isothermal flows. On the other hand, it is worth noting that the above incompressible GKS showed some deficiencies at high Reynolds numbers, such as artificial oscillation in the pres-

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sure distribution [18]. As indicated by Chen et al. [20], the assumption of smooth distribution of flow variables and their derivatives at the cell interface may destroy the robustness of incompressible GKS at high Reynolds numbers. To resolve this problem, they proposed a GKS with discontinuous derivatives at the cell interface, i.e. the scheme with weak discontinuity, for simulation of incompressible isothermal flows. Their scheme can effectively improve the numerical stability of incompressible GKS, while its implementation is almost as complicated as the compressible one.

Also aiming at the calculation of numerical fluxes by the local solution of Boltzmann equation, a series of simplified gas kinetic schemes are proposed by Shu and his coworkers [21–23]. For the three-dimensional (3D) case, the simplified GKS is termed as sphere function-based GKS (SGKS) [22,23]. There are mainly two improvements of SGKS as compared with the Maxwellian function-based GKS [1,2,11–15]. The first one is to replace the Maxwellian distribution function by a simple sphere function so that the integrals for conservation forms of moments in the infinity domain for the Maxwellian function-based GKS can be reduced to those in the finite domain (integrals along the spherical surface) for the SGKS. The second one is to use the difference of equilibrium distribution functions at the cell interface and its surrounding points to approximate the non-equilibrium distribution function so that the local solution of Boltzmann equation can be expressed in an algebraic form. These simplifications make the computation of SGKS be simpler and more efficient than the corresponding Maxwellian function-based GKS. On the other hand, the formulations of SGKS are determined by the integral domain along the spherical surface, which is located at (u, v, w) with radius c, where (u, v, w) is the macroscopic flow velocity and c is proportional to the sound speed. For compressible flows, the sphere is usually not symmetric at the cell interface. This leads the calculation of numerical fluxes at the cell interface for the SGKS to be relatively complicated as compared with the conventional Navier–Stokes solvers [24,25]. However for incompressible flows, the sphere at the cell interface can be approximately considered to be symmetric as the flow velocity is far less than the sound speed. Besides that, the energy equation is usually not needed for simulation of incompressible isothermal flows as shown in the works of Xu and He [18] and Guo et al. [19]. Based on these facts, a simplified SGKS is developed to improve the computational efficiency for simulation of incompressible flows in this work. In the method, the formulations of numerical fluxes at the cell interface can be expressed concisely and explicitly. In addition, like the original SGKS, the discontinuity of conservative variables and their derivatives at the cell interface are still kept in the simplified SGKS so as to keep good numerical stability at high Reynolds numbers.

For solving fluid flow problems, the time-implicit scheme has achieved great advancements due to its high efficiency and stability. In the implicit scheme, the approximate factorization is usually adopted to get the linearized Jacobian matrix and the iterative method is applied to solve the derived linear system. One of the widely-used iterative methods is the matrix-free Lower-Upper Symmetric Gauss-Seidel (LU-SGS) [26–28] method. In this method, the derived linear system can be solved directly by a forward and a backward sweep. Thus, the LU-SGS method requires little computational effort compared to other implicit schemes, such as the Alternating Direction Implicit (ADI) scheme [29,30]. Due to its simplicity, the LU-SGS method has been used in GKS for simulation of both continuous flow [13,31–34] and rarefied flow [35,36]. In the work of Xu et al. [13], a LU-SGS method-based implicit GKS was developed for simulation of hypersonic viscous flow. By using the implicit GKS and kinetic boundary conditions, Li and Fu [31] studied two typical flows in the near continuum regime, i.e., the hypersonic flow around a hollow cylinder flare and the flow in microchannels. Jiang and Qian [32] applied the implicit GKS and multi-grid technique to simulate 3D stationary transonic high-Reynolds number flows. In the work of Li et al. [33], the implicit GKS on unstructured meshes was developed for solving high temperature equilibrium gas flows. Moreover, Tan and Li [34] compared the performance of implicit GKS with the LU-SGS method and the preconditioned generalized minimum residual (GMRES) method. Note that in all the above implicit GKS, the Maxwellian distribution function is adopted for calculation of the numerical fluxes at the cell interface. In addition, they are mainly used for simulation of compressible flows. In this work, the commonly-used LU-SGS method is introduced into the simplified SGKS to simulate 3D incompressible isothermal flows. The combined scheme is called as implicit simplified SGKS in the following text. The computational accuracy and efficiency of the developed scheme are validated by simulating the 3D lid-driven cavity flow and the flow over a backward-facing step.

2. Incompressible Navier–Stokes equations, Boltzmann equation and sphere function

2.1. Incompressible Navier–Stokes equations and FVM discretization

The macroscopic governing equations based on mass and momentum conservation laws for incompressible isothermal flows can be written in a weakly compressible form as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{1}$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u} + p\mathbf{I}) = \nabla \cdot \left\{ \mu \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right] \right\} \tag{2}$$

where ρ, $\mathbf{u}$, p and μ are respectively the density, velocity, pressure and dynamic viscosity of fluid flow. $\mathbf{I}$ is the unit tensor.

As the simplified sphere function-based gas kinetic scheme (SGKS) will be applied to compute the numerical fluxes at the cell interface, we prefer to define the conservative variables at cell centers and discretize the governing Eqs. (1) and (2) by finite volume method (FVM). For the 3D case, Eqs. (1) and (2) discretized by FVM can be written as

$$\frac{d(\Omega_I \mathbf{W}_I)}{dt} = -\sum_{i=1}^{N_f} \mathbf{F}_{ni} S_i \tag{3}$$

where I is the index of a control volume, Ω_I and N_f represent the volume and the number of the faces of the control volume I, respectively. S_i denotes the area of the ith face of the control volume. The conservative variable vector $\mathbf{W}$ and flux vector $\mathbf{F}_n$ are given by,

$$\mathbf{W} = (\rho, \rho u, \rho v, \rho w)^T \tag{4}$$

$$\mathbf{F}_n = \left(F_\rho, F_{\rho u}, F_{\rho v}, F_{\rho w} \right)^T \tag{5}$$

where $\mathbf{u} = (u, v, w)$ is the velocity vector expressed in the global Cartesian coordinate system. There are two keys for solving Eq. (3). One is the numerical method for calculation of fluxes $\mathbf{F}_n$, and the other one is the iterative algorithm for updating the conservative variables $\mathbf{W}$. They will be discussed in Sections 3 and 4, respectively.

For the convenience of derivation, the local coordinate system defined at the cell interface is introduced in this work. In the local coordinate system, x_1-axis is taken as the normal direction pointing always outwards of the cell interface, x_2-axis and x_3-axis are chosen as two tangential directions on the cell interface, which are mutually orthogonal. The conservative variables and fluxes expressed in the local coordinate system can be written as

$$\overline{\mathbf{W}} = (\rho, \rho u_1, \rho u_2, \rho u_3)^T \tag{6}$$