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Development of high-order weighted compact schemes with various difference methods

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ABSTRACT

This paper is concerned with the development of high-order weighted compact schemes based on linear central compact schemes (Liu et al., 2013). For the purpose of capturing strong discontinuities, the high-order upwind weighted nonlinear interpolations are combined with the central compact schemes and cell-centered compact schemes to achieve two different weighted compact schemes, called the CCSSR-W and CCCSSR-W schemes respectively. The CCSSR-W schemes are developed up to ninth order, and four different nonlinear weights of CCSSR-W schemes are investigated in detail. As an improvement of the CCSSR-HW scheme (Liu et al., 2015), the hybrid weighted nonlinear interpolation is substituted into the cell-centered compact scheme to obtain the CCCSSR-HW scheme. For all the proposed schemes, the flux vector splitting method is carried out by coupling with characteristic projections on the conservative fluxes. Through the systematic accuracy tests, spectral analysis and numerical experiments, the results demonstrate that the higher order CCSSR-W scheme is more accurate and effective than the lower order ones. The CCCSSR-W scheme has better accuracy and resolution than the original WENO scheme. The CCCSSR-HW scheme has higher resolution and lower dissipation than the CCSSR-HW scheme, and even better than the higher order CCSSR-W schemes.

1. Introduction

The high-order numerical methods have low dissipation and dispersion errors, which are especially suitable for complex hydrodynamic problems, such as the compressible turbulence. Weighted essentially non-oscillatory (WENO) scheme is a quite popular high-order shock capturing method, which has been widely used for the compressible flows. Liu, Osher and Chan [14] first proposed the finite volume WENO scheme based on essentially non-oscillatory (ENO) scheme [9,25,26], using the nonlinear weights to achieve the higher order of accuracy. In 1996, Jiang and Shu [11] provided a general framework to construct arbitrary order accurate finite difference WENO schemes, which were more efficient for multi-dimensional calculations. Balsara and Shu [1] developed the WENO schemes up to the eleventh-order using the monotonicity-preserving procedure of Suresh and Huynh [28]. Shi et al. [23] performed a numerical study on the resolution of high-order WENO finite difference methods for several benchmark problems containing both discontinuities and complex fluid structures.

However, Fedkiw et al. [7] noted that ϵ, a parameter in WENO scheme used for avoiding a division by zero in the denominator, was a dimensional quantity. They pointed out that large value of ϵ caused the WENO scheme toward linear difference scheme, while small value of ϵ might cause the WENO scheme to reduce the order in the presence of critical points. Subsequently, Henrick et al. [10] devised a mapping function and applied it to the nonlinear weights of classical WENO scheme [11]. The improved fifth-order WENO scheme (WENO-M) can achieve the optimal convergence order at critical points even with very small ϵ, at the cost of increasing computational time. Borges et al. [2] introduced another version of the fifth-order WENO scheme (WENO-Z) with a global optimal smoothness indicator, which was the absolute value of linear combination of the lower order local smoothness indicators. This method could achieve the optimal convergence order with very small ϵ at the critical points. In their following papers [4,6], the authors developed higher order WENO-Z schemes up to the thirteenth-order. In [31], Zhao et al. evaluated the improved fifth-order WENO schemes [2,10] for the implicit large eddy simulation of turbulent flows, they found that the WENO-Z scheme resulted in

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a drastically reduced computational cost compared with WENO-M scheme.

Although WENO schemes provide sharp shock-capturing, they underestimate most of the resolvable wavenumbers for broadband problems due to high numerical dissipation [12]. To simulate the multi-scale problems, such as turbulence and acoustic wave, a good choice is the linear compact scheme, which has better resolution than explicit finite difference scheme at the same order of accuracy. The most influential compact scheme for derivative, interpolation and filtering was proposed by Lele [13]. Through systematic Fourier analysis, it is shown that these compact schemes have spectral-like resolution for short waves. In [15], Liu et al. developed a class of linear central compact schemes, which have better accuracy and resolution than the cell-centered compact schemes of Lele [13]. Linear compact schemes have been widely used for wave propagation problems in which nonlinearities are weak. However, it is well known that application of central discretization to high-Reynolds number fluid typically leads to numerical instability, and linear compact scheme can not deal with strong discontinuities.

The combination of WENO scheme [11] and cell-centered compact scheme [13] has been developed by several researchers. Deng and Zhang [5] proposed the weighted compact nonlinear scheme (WCNS) for conservative variables, using the flux difference splitting method to achieve the numerical fluxes. Nonomura et al. [19] developed the WCNS schemes [5] up to the seventh- and ninth-order. In [17], Nonomura and Fujii investigated the effects of difference scheme type for high-order WCNS schemes [5,19]. These results show that the explicit difference scheme is preferable, and the weighted nonlinear interpolation is dominant for the resolution of WCNS scheme. Instead of interpolating conservative variables, Zhang et al. [30] developed a class of high-order weighted nonlinear interpolations for conservative fluxes. Through coupling with the cell-centered compact schemes [13], they proposed the weighted compact (WCOMP) schemes, which have slightly higher resolution than classical WENO schemes [11] at the same order. In [18], Nonomura and Fujii proposed a robust explicit weighted nonlinear scheme (WCNS-MND), in which the linear difference scheme just as the explicit type of linear compact scheme by Liu et al. [15], and the weighted nonlinear interpolation procedure was used for conservative variables. They found that this scheme was more robust and dissipative than WCNS scheme [5,17,19]. Recently, Liu et al. [16] proposed hybrid weighted compact schemes (CCSSR-HW) for conservative fluxes based on the hybrid factor of Ren et al. [21], these schemes have higher resolution and lower dissipation than WENO schemes, and perform good shock-capturing properties. However, the CCSSR-HW schemes [16] are even order (fourth-order and sixth-order), this strategy is a little difficult to be developed up to higher order schemes. In addition, there are some empirical parameters in the hybrid factor [21], which are needed to adjust for different numerical experiments.

In this paper, we propose new weighted compact schemes with essentially non-oscillatory behavior across discontinuities, which are based on the linear central compact schemes proposed by Liu et al. [15]. The high-order weighted nonlinear interpolations, including upwind and hybrid interpolations, are implemented for two types of linear compact schemes, then the flux vector splitting method is used to acquire the numerical conservative fluxes on the cell-centers. We also study the performance of four different nonlinear weights for our new weighted compact schemes and WENO schemes. For all numerical experiments, the comparisons between the proposed schemes and several representative nonlinear schemes are made for the resolution and dissipation. Through systematic analysis, numerical tests and comparisons, we show that our new weighted compact schemes are high order, high spectral resolution and low dissipation errors, while have good behavior to capture strong discontinuities.

This paper is organized as follows. The numerical methodology is presented in Section 2, including the linear central compact schemes and weighted nonlinear compact schemes. The numerical accuracy tests are performed in Section 3. The systematic analysis for the dispersion and dissipation characteristics of the proposed schemes is shown in Section 4. Numerical experiments including strong discontinuities are given in Section 5. The conclusions are stated in Section 6.

2. Numerical methodology

In this section, the numerical methodology for our new weighted compact scheme is presented.

We consider the numerical solution of the conservation law

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0$$

A semi-discrete finite difference can be represented as

$$\left(\frac{\partial u}{\partial t}\right)_j = -f'_j$$

where f'_j is the approximation to the spatial derivative $\frac{\partial f(u)}{\partial x}$ at the grid node x_j.

We start this work from the linear central compact schemes proposed by Liu et al. [15], then extend these linear schemes to high-order weighted nonlinear compact schemes with essentially non-oscillatory behavior across discontinuities.

2.1. Linear central compact schemes

In [13], Lele proposed the linear cell-centered compact schemes with spectral-like resolution (CCCSSR), which have the following form

$$\beta f'_{j-2} + \alpha f'_{j-1} + f'_j + \alpha f'_{j+1} + \beta f'_{j+2} = a\frac{f_{j+\frac{1}{2}} - f_{j-\frac{1}{2}}}{\Delta x} + c\frac{f_{j+\frac{3}{2}} - f_{j-\frac{3}{2}}}{3\Delta x} + e\frac{f_{j+\frac{5}{2}} - f_{j-\frac{5}{2}}}{5\Delta x} \tag{2.1}$$

Using both the values on the cell-nodes and cell-centers, Liu et al. [15] proposed the linear central compact schemes with spectral-like resolution (CCSSR), which improved the CCCSSR schemes [13] by

$$\beta f'_{j-2} + \alpha f'_{j-1} + f'_j + \alpha f'_{j+1} + \beta f'_{j+2} = a\frac{f_{j+\frac{1}{2}} - f_{j-\frac{1}{2}}}{\Delta x} + b\frac{f_{j+1} - f_{j-1}}{2\Delta x} + c\frac{f_{j+\frac{3}{2}} - f_{j-\frac{3}{2}}}{3\Delta x} + d\frac{f_{j+2} - f_{j-2}}{4\Delta x} + e\frac{f_{j+\frac{5}{2}} - f_{j-\frac{5}{2}}}{5\Delta x} \tag{2.2}$$

The relationships among the coefficients of Eqs. (2.1) and (2.2) are derived by matching the Taylor series of various orders. The accuracy of CCCSSR scheme ranges from second-order to tenth-order. However, the accuracy of CCSSR scheme ranges from second-order to fourteenth-order. The detailed coefficients of the CCSSR and CCCSSR schemes are presented in [15].

If the schemes are restricted to $\alpha = 0$ and $\beta = 0$, a family of explicit CCSSR and CCCSSR schemes are obtained. If the schemes are restricted to $\alpha \neq 0$ and $\beta = 0$, a variety of tridiagonal CCSSR and CCCSSR schemes are obtained. If $\alpha \neq 0$ and $\beta \neq 0$, pentadiagonal CCSSR and CCCSSR schemes are generated. Taking the CCSSR schemes for instance, three kinds of schemes are denoted by CCSSR-E, CCSSR-T and CCSSR-P respectively.

The tridiagonal schemes have the best combinations of resolution characteristics, order of accuracy and computational efficiency. Therefore, in this paper, we just focus on the tridiagonal schemes. The accuracy of tridiagonal CCSSR scheme ranges from