Computers and Fluids 136 (2016) 324–330

Contents lists available at ScienceDirect

Computers and Fluids

journal homepage: www.elsevier.com/locate/compfluid

A novel well-balanced scheme for modeling of dam break flow in drying-wetting areas

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ARTICLE INFO

Article history:
Received 22 January 2016
Revised 24 June 2016
Accepted 26 June 2016
Available online 27 June 2016

Keywords:
Dam break
Finite element method
Characteristic-Galerkin procedure
Well-balanced
Drying-wetting

ABSTRACT

A well-balanced explicit/semi-implicit finite element scheme is proposed for the simulation of dam break flows over complex domains involving wetting and drying. The numerical model is based on the nonlinear shallow water equations in the hyperbolic conservation form. The governing equations are discretized by a fractional finite element method using a characteristic-Galerkin procedure. Firstly, the intermediate increment of a conserved variable is obtained explicitly neglecting the pressure gradient term. And then, the increment is corrected for the effects of pressure once the pressure increment is obtained from the Poisson equation. In order to maintain the "well-balanced" property, the pressure gradient term and bed slope terms are incorporated into the Poisson equation. Moreover, a local bed slope modification technique is employed in drying-wetting interface treatments. The new model is validated against several benchmark tests and laboratory experimental datas related to dam-break flood wave propagation and promising results are obtained.

1. Introduction

Dam break flows could lead to severe flooding with catastrophic consequences, such as damage to properties and loss of human life. Examples of such incidents include the failure of the Tenton dam in 1976 and the levee failures in the Mississippi River in 1993 and in the Yangtze River in 1998. In recent years, it has become compulsory to set up emergency plans for dam breaks. Numerical modeling and understanding of dam break flows are playing an increasingly essential role in hydraulic and river engineering in the aspect of reservoir safety.

The shallow water equations(SWEs) give a surprisingly accurate representation of the dam-break flood due to its long wave hydrodynamics and the neglectable vertical acceleration of water particles. In literature, there are many numerical methods for dam break flows, including the finite difference method (FDM) [1,2], the finite volume method (FVM) [3,4], and the discontinue Galerkin (DG) method [5,6]. As an alternative, the finite element method (FEM) [7,8] has also been successfully employed in numerous applications due to its strict mathematical foundation and capability of handling complex geometries and boundary conditions exactly. However, two issues relevant in many applications are still under investigation: namely (i) preserving steady-states with variable ground elevation and (ii) properly handling drying and wetting [9].

The first issue related in numerical simulation of SWEs is the so-called 'C-property', or 'well-balanced property' which is first proposed by Bermúdez and Vázquez [10,11]. Essentially, a numerical imbalance is created by artificially splitting the surface gradient into flux gradient and bed slope terms, and improper discretization of these terms may not result in a well-balanced scheme [3]. To preserve the still water or well-balanced property of the scheme, modified shallow water equations [12–14], flux modification [9,15], exact Riemann solver [4],and surface gradient methods [16] can be used. Although there are extensive literatures on the "well-balanced" property for both the FVM and DG, a few such treatments for the FEM solutions have been reported. This will be one of the topics of the presented paper.

Another major issue that has to be involved when solving problems governed by the SWEs is the handling of drying and wetting. If no special attention is paid, standard numerical methods may fail near a drying-wetting front and may produce an unacceptably negative water height [15]. Fruitful results about this topic have been achieved recently [5,17–22]. A more popular approach is the thin layer technique [5,18,19], which maintains a very thin layer of dry elements included in the computation. An advantage of the thin layer element approach is that mass can be conserved because elements maintain full connectivity within the continuity or conservation of water volume equation. Furthermore, water is kept

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http://dx.doi.org/10.1016/j.compfluid.2016.06.022
0045-7930/

in these nominally dry elements until they are restored as fully wet. On the other hand, conservation of momentum is not guaranteed due to the erroneous water depth gradient in case of the real drying-wetting interface is not being located at the node of an element exactly. P. Brufau [23] proposed the idea of locally modifying the bottom level of the dry node so that a horizontal water level is achieved. The drying-wetting treatment presented here adopts the thin water layer technique so that it can achieve a good balance between accuracy and computational cost. And moreover, similar to P. Brufau [23] but quite differently, a depth gradient modification approach is proposed to deal with the momentum conservation problem.

The purpose of this work is to develop a new well balanced explicit/semi-implicit finite element scheme for dam break flows simulation in drying-wetting areas. In summary, the main features of the new scheme are: (i) both the explicit and semi-implicit FEM scheme is implemented (ii) it has the 'well-balanced property' over a non-flat bed, and (iii) the ability to handle the drying-wetting problem. These features are demonstrated using several benchmark problems for dam break flows. The results presented in this paper show the accuracy of the proposed scheme and confirm its capability to provide accurate and efficient simulations for shallow water flows including complex topography.

This paper is organized as follows. The governing equations are briefly presented in next section. The explicit/semi-implicit finite element scheme is formulated in detail in Section 3. The source balance and drying-wetting treatment are discussed in this section. Several challenging test cases are reported to validate the presented scheme in Section 4. Finally, concluding remarks are summarized in Section 5.

2. Governing equations

The two-dimensional depth-averaged SWEs, which can be derived from integrating the Navier–Stokes equations over the flow depth by assuming a hydrostatic pressure distribution, are widely used for governing the dam-break flows. The conserved form of SWEs in Cartesian coordinates are

$$\frac{\partial h}{\partial t} + \nabla \cdot \boldsymbol{U} = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{U}}{\partial t} + \nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{U}) = \nabla \cdot \boldsymbol{\tau} - \nabla\left(\frac{1}{2}gh^2\right) + \boldsymbol{S}_b + \boldsymbol{S} \tag{2}$$

Where t is the time, ∇ is the horizontal gradient operator, $\boldsymbol{U} = \boldsymbol{u}h$ is the conserved variable, $\boldsymbol{u}$ is the depth-averaged horizontal velocity, $\boldsymbol{\tau} = h\nu(\nabla\boldsymbol{u} + \nabla^T\boldsymbol{u})$ is viscous term, ν is the kinematic viscosity coefficient, g is the gravity acceleration and h is the total depth of water. Here the source term is divided into two parts: the bed slope term $\boldsymbol{S}_b = -gh\nabla z$ and the remaining terms S, where z is the elevation of bottom. For the sake of simplicity and without loss of generality, the contribution to the source term from the bottom friction, Coriolis force, the wind tractions and the atmospheric pressure gradients are all included in S.

3. Numerical scheme

Defining the wave speed $c = \sqrt{gh}$, and introducing two time parameters, θ_1, θ_2, the governing equations are rewritten in the following time discretized form:

$$\frac{1}{c^2}\frac{\Delta p}{\Delta t} + \nabla \cdot \boldsymbol{U}^{n+\theta_1} = 0 \tag{3}$$

$$\frac{\Delta \boldsymbol{U}}{\Delta t} + \nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{U}) = \nabla \cdot \boldsymbol{\tau} - \nabla p^{n+\theta_2} + \boldsymbol{S}_b + \boldsymbol{S} \tag{4}$$

Here the parameter θ_1 and θ_2 can be chosen in the range [0,1]. $1/2 \le \theta_1 \le 1$, $1/2 \le \theta_2 \le 1$ for the semi-implicit form, and $1/2 \le \theta_1 \le 1, \theta_2 = 0$ for an explicit form. The pseudo-pressure p is defined as $p = gh^2/2$ to maintain the analogy with equations of compressible flows. In fact, the SWEs are the hydraulic analogy of the Euler equations, which govern gas dynamics [24]. The analogy is rooted in the correspondence between the water elevation in the SWEs and the density in the gas dynamic equations. Given that the Euler differential system for gas dynamics admits discontinuous solutions, called shocks and contact discontinuities, the existing analogy implies the possibility of hydraulic jumps and bores in water and the propagation of a sharp front in the atmosphere. Thus, it is tempting to apply the successful methods developed for gas dynamics problems to hydraulic governed by the shallow water equations.

3.1. Numerical discretized using characteristic-Galerkin procedure

The time discretization of Eq. (4) along the characteristic gives

$$\Delta \boldsymbol{U} = -\Delta t\left(\nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{U}) - \nabla \cdot \boldsymbol{\tau}^T - \boldsymbol{S}\right) - \Delta t\left(\nabla p^{n+\theta_2} - \boldsymbol{S}_b\right) + \frac{\Delta t^2}{2}\boldsymbol{u} \cdot \nabla(\nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{U}) - \boldsymbol{S}) + \frac{\Delta t^2}{2}\boldsymbol{u} \cdot \nabla\left(\nabla p^{n+\theta_2} - \boldsymbol{S}_b\right) \tag{5}$$

Following the fractional step procedure proposed by Chorin [25], the increment $\Delta \boldsymbol{U}$ can be decomposed into two parts:

$$\Delta \boldsymbol{U} = \Delta \boldsymbol{U}^* + \Delta \boldsymbol{U}^{**} \tag{6}$$

$$\Delta \boldsymbol{U}^* = -\Delta t\left(\nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{U}) - \nabla \cdot \boldsymbol{\tau}^T - \boldsymbol{S}\right) + \frac{\Delta t^2}{2}\boldsymbol{u} \cdot \nabla(\nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{U}) - \boldsymbol{S}) \tag{7}$$

$$\Delta \boldsymbol{U}^{**} = -\Delta t\left(\nabla p^{n+\theta_2} - \boldsymbol{S}_b\right) + \frac{\Delta t^2}{2}\boldsymbol{u} \cdot \nabla\left(\nabla p^{n+\theta_2} - \boldsymbol{S}_b\right) \tag{8}$$

Firstly, the increment of intermediate variable $\Delta \boldsymbol{U}*$ is obtained explicitly by omitting the pressure gradient and bed slope terms. $\boldsymbol{U}$ is then corrected for the effects of pressure once the pressure increment Δp^n has been obtained from the Poisson equation implicitly or explicitly. In this way, we get the increment$\Delta \boldsymbol{U}**$, and also $\Delta \boldsymbol{U}$ at last.

It is worthy to point out that in order to obtain a well-balanced scheme, the bed slope source term should not be included in first step, but is incorporated in the pressure Poisson equation, and it will be illustrated in Section 3.2.

3.1.1. Discretization for increment of intermediate variable $\Delta \boldsymbol{U}^*$

Firstly, the increment of intermediate variable $\Delta \boldsymbol{U}*$ is obtained from Eq. (7), in which both the pressure gradient term and the bed slope source term are omitted, by using the characteristic-Galerkin scheme.

Following the standard Galerkin approximation and approximating spatially using standard finite element shape functions, spatial discretization of Eq. (7) gives

$$\boldsymbol{M}\Delta\bar{\boldsymbol{U}}^* = \boldsymbol{RHS}_1 \tag{9}$$

where $\boldsymbol{M} = \int \boldsymbol{N}^T\boldsymbol{N}d\Omega$,

$$\boldsymbol{RHS}_1 = -\Delta t\left[\int \boldsymbol{N}^T[\nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{U}) - \boldsymbol{S}]d\Omega + \int \nabla\boldsymbol{N}^T \cdot \boldsymbol{\tau}^T d\Omega\right] - \frac{\Delta t^2}{2}\int \nabla \cdot \left[\boldsymbol{N}^T\boldsymbol{u}\right][\nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{U}) - \boldsymbol{S}]d\Omega + \Delta t\int \boldsymbol{N}^T\boldsymbol{\tau}^T \cdot \vec{n}d\Gamma$$

The overbar of $\Delta\bar{\boldsymbol{U}}^*$ represents the approximate nodal values and $\boldsymbol{N}$ is the shape functions for the field $\boldsymbol{U}$.

3.1.2. Continuity equation discretization

As in the fractional step procedure, the present method computes the pressure p (or elevations of the free surface h) by