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Numerical modeling of non-Newtonian biomagnetic fluid flow

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ABSTRACT

Blood flow dynamics have an integral role in the formation and evolution of cardiovascular diseases. Simulation of blood flow has been widely used in recent decades for better understanding the symptomatic spectrum of various diseases, in order to improve already existing or develop new therapeutic techniques. The mathematical model describing blood rheology is an important component of computational hemodynamics. Blood as a multiphase system can yield significant non-Newtonian effects thus the Newtonian assumption, usually adopted in the literature, is not always valid. To this end, we extend and validate the pressure correction scheme with discontinuous velocity and continuous pressure, recently introduced by Botti and Di Pietro for Newtonian fluids, to non-Newtonian incompressible flows. This numerical scheme has been shown to be both accurate and efficient and is thus well suited for blood flow simulations in various computational domains. In order to account for varying viscosity, the symmetric weighted interior penalty (SWIP) formulation is employed for the discretization of the viscous stress tensor. We disregard the dependency of the viscosity on spatial derivatives of the velocity in the Jacobian computation. Even though this strategy yields an approximated Jacobian, the convergence rate of the Newton iteration is not significantly affected, thus computational efficiency is preserved. Numerical accuracy is assessed through analytical test cases, and the method is applied to demonstrate the effects of magnetic fields on biomagnetic fluid flow. Magnetoviscous effects are taken into account through the generated additive viscosity of the fluid and are found to be important. The steady and transient flow behavior of blood modeled as a Herschel-Bulkley fluid in the presence of an external magnetic field, is compared to its Newtonian counterpart in a straight rigid tube with a 60% axisymmetric stenosis. A break in flow symmetry and marked alterations in WSS distribution are noted.

1. Introduction

In recent years significant research work has been directed towards studying the effects of magnetic fields on biomagnetic fluid flow, with ample applications in bioengineering and the medical sciences [1,2]. The most common biofluid is blood which behaves as a magnetic fluid because of the hemoglobin molecule that is present in red blood cells. To this end, Haik et al. [3] developed a Biomagnetic Fluid Dynamics (BFD) model by considering the Langevin equation for the magnetization of classical fluids. Haik's model does not take into account the electric properties of biofluids. As a result, magnetic effects appear solely in terms of the field gradients generating a corresponding magnetization force. Experiments on cow and sheep blood though have shown appreciable dielectric properties for blood [4], which produces a Lorentz force even in the case of constant magnetic fields [5]. The idea of considering the magnetic and electric properties of blood under a unified mathematical model was introduced by Tzirtzilakis [6]. To this end, both forces were included in the Navier–Stokes equations, where blood was assumed to behave as a Newtonian fluid [7–10].

The assumption of Newtonian behavior for blood has been widely used in the literature. Even though it may be valid for flows that are characterized by shear rates higher than 100 s^{-1} where deviations from the Newtonian behavior may be small [11], the Newtonian hypothesis becomes problematic for lower shear rates. In addition, during the end-deceleration phase of pulsatile cycles, shear rates can decline to values lower than the 100 s^{-1} limit, generating potentially important non-Newtonian effects. This deviation becomes even more profound in small arteries and veins or in numerous diseased conditions [12,13].

The transition from the Newtonian to a non-Newtonian consideration for biofluids such as blood is accompanied by a choice

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for the mathematical model describing it. There are many models for the characterization of non-Newtonian blood behavior. Casson [14,15] and Herschel–Bulkley [16] are the models amongst others (Power-law, Carreau, Bingham) that appear most often in literature. The Herschel–Bulkley compared to the Casson fluid model though has two distinctive advantages. For arterioles with diameters less than 0.065 mm, the Casson model does not capture velocity profiles accurately [17]. In addition, the constitutive equation of the Herschel–Bulkley model has two degrees of freedom (instead of one for the Casson model) yielding a better description of blood flow under a wider range of realistic conditions.

The combined goal of accurately describing blood behavior and of altering its flow using externally applied magnetic fields can be applied to targeted drug deposition, yielding higher drug concentration at specific sites and reducing total required dosage. This can be achieved by injecting magnetic nanoparticles into blood flow which interact with magnetic fields around a desired target location [18]. In addition, external magnetic fields will alter viscosity due to generated magnetoviscous effects. Specifically, it has been shown by Haik et al. [19] that a static magnetic field of 4T can increase the apparent viscosity of human blood by approximately 11.5%. It is thus important to account for magnetoviscous effects on blood flow exposed to external magnetic fields.

The available literature on non-Newtonian flows is expanding. Starting with Shukla et al. [20] many researchers have studied non-Newtonian flows in arterial stenosis [12,13,21,22]. More recently, Kröner et al. [23] presented a fully implicit Local Discontinuous Galerkin (LDG) discretization for non-Newtonian incompressible flows. Due to the computational expense of the scheme though they considered 2D computations only. Recently, Kwack and Masud [24] presented a stabilized mixed FEM to non-Newtonian shear-rate dependent flows, where viscosity is considered a nonlinear function of shear-rate. Along these lines, they developed a stabilized numerical scheme using the Variational Multiscale framework to the underlying generalized Navier–Stokes equations. Here we employ the pressure correction formulation proposed by Botti and Di Pietro [25], which has demonstrated to be effective for high-Reynolds hemodynamic simulations in real patient geometries [26]. Specifically, pressure gradients in hemodialysis patients were simulated and compared with experiments for various steady conditions and wide range of Reynolds numbers (100–2000). The results closely followed the experimental data, yielding an accurate solver for convection-dominated incompressible flows.

In this study, we consider the Symmetric Weighted Interior Penalty (SWIP) formulation for the stress tensor, ignoring the dependence of viscosity on the velocity solution in the Jacobian computation. The resulting approximated Jacobian does not alter the convergence rate of the Newton iteration significantly, retaining the computational efficiency unchanged. The novelty of the proposed scheme is a fast and accurate algorithm for realistic blood flow simulations where in some cases the computational domain consists of hundreds of thousands if not millions of elements. To the authors knowledge, this is the first numerical simulation study of non-Newtonian biomagnetic fluid flow using the SWIP formalism, and is a generalization of a previous work on Newtonian biofluid flow exposed to external magnetic fields [27], to biofuids that are characterized by non-constant viscosity.

The paper is organized as follows: Section 2 analyzes the mathematical framework for the laminar flow of a non-Newtonian fluid in the presence of external magnetic fields. Section 3 describes the numerical implementation used for the solution of the Navier–Stokes equations. Section 4 examines the numerical validation of the proposed method and Section 5 contains the results and comparison with exact solutions when these are available. Finally, Section 6 presents the summary and conclusions.

Table 1
Flow models for various shear rate dependent viscosities.

$\mu = \kappa$	Newtonian
$\mu(\dot{\gamma}) = \kappa\dot{\gamma}^{n-1}$	Power-law
$\mu(\dot{\gamma}) = (\tau_0/\dot{\gamma})[1-\exp(-m\dot{\gamma})] + \kappa\dot{\gamma}^{n-1}$	Herschel–Bulkley

2. General setting

2.1. Flow model

Let $\Omega \subset \mathbb{R}^d$, $d = 2, 3$, denote a bounded, connected open set, and let $t_F > 0$ denote the final simulation time. We consider the unsteady incompressible Navier-Stokes equations with Dirichlet and traction-free outflow boundary conditions to be imposed on the domain boundaries $\partial\Omega_D$ and $\partial\Omega_N$ respectively,

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho\mathbf{u}\cdot\nabla\mathbf{u} - \nabla\cdot\boldsymbol{\sigma} = \rho\mathbf{f} \qquad \text{in } \Omega\times(0,t_F), \tag{1a}$$

$$\nabla\cdot\mathbf{u} = 0 \qquad \text{in } \Omega\times(0,t_F), \tag{1b}$$

$$\mathbf{u} = \mathbf{g}_D \qquad \text{on } \partial\Omega_D\times(0,t_F), \tag{1c}$$

$$\boldsymbol{\sigma}\cdot\boldsymbol{n} = \mathbf{h} \qquad \text{on } \partial\Omega_N\times(0,t_F), \tag{1d}$$

$$\mathbf{u}(\cdot, t=0) = \mathbf{u}_0 \qquad \text{in } \Omega, \tag{1e}$$

where u_0 is the initial condition, $\mathbf{g}_D$ is the Dirichlet velocity boundary condition, $\mathbf{h}$ is the prescribed boundary traction vector, ρ and $\mathbf{f}$ are the density, and body force respectively. The homogeneous expression of (1d) corresponds to the traction-free boundary condition which is widely imposed as artificial outflow boundary. Additionally, $\boldsymbol{\sigma}$ is the stress tensor given by the following expression,

$$\boldsymbol{\sigma}(\mathbf{u}, p) = -p\mathbf{I} + \boldsymbol{\tau}(\mathbf{u}). \tag{2}$$

The shear-stress tensor, $\boldsymbol{\tau}(\mathbf{u})$, is written in terms of the deformation rate tensor, $\mathbf{D}(\mathbf{u}) \equiv \frac{1}{2}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right]$, and its three invariants as,

$$\boldsymbol{\tau}(\mathbf{u}) = 2\mu(\mathbf{D})\mathbf{D}(\mathbf{u}) = 2\mu(I_{\mathbf{D}(\mathbf{u})}, II_{\mathbf{D}(\mathbf{u})}, III_{\mathbf{D}(\mathbf{u})})\mathbf{D}(\mathbf{u}). \tag{3}$$

Since the flow is incompressible ($I_{\mathbf{D}(\mathbf{u})} = 0$) and assuming the third invariant is negligible for shear flows we find that,

$$\boldsymbol{\tau}(\mathbf{u}) = 2\mu(II_{\mathbf{D}(\mathbf{u})})\mathbf{D}(\mathbf{u}). \tag{4}$$

Defining finally shear rate $\dot{\gamma} \equiv 2\sqrt{II_{\mathbf{D}(\mathbf{u})}}$, the magnitude of the shear-stress tensor takes the form,

$$\tau = \mu(\dot{\gamma})\,\dot{\gamma}, \tag{5}$$

where in Cartesian coordinates,

$$\dot{\gamma}^2 = 2\left(\frac{\partial u_x}{\partial x}\right)^2 + 2\left(\frac{\partial u_y}{\partial y}\right)^2 + 2\left(\frac{\partial u_z}{\partial z}\right)^2 + \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)^2 + \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_w}{\partial x}\right)^2 + \left(\frac{\partial u_y}{\partial z} + \frac{\partial u_w}{\partial y}\right)^2. \tag{6}$$

The different expressions of the shear rate dependent viscosity models that are considered in this paper are presented in Table 1. The simplest non-Newtonian constitutive equation yields the so-called power-law model which is characterized by two parameters. The consistency index, κ, and power index, n. The Newtonian case is then simply obtained by setting $\mu = \kappa$ and $n = 1$. For blood flow simulations, the Herschel–Bulkley model is considered. Due to discontinuity though at yield stress, τ_0, the generalization proposed by Papanastasiou [28] as expressed by the exponential term and regularization parameter, m, is also included. As such, the above models are chosen on the basis of being effective in providing good fits to blood viscosity