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Noise reduction in car aerodynamics using a surrogate objective function and the continuous adjoint method with wall functions

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ABSTRACT

The purpose of this paper is twofold. In the first part, the continuous adjoint formulation for field integral objective functions used in steady-state, incompressible aerodynamic optimization is developed. The formulation includes the full differentiation of the Spalart–Allmaras turbulence model based on wall functions. In the second part, the developed adjoint method is used for optimizing the side mirror shape of a passenger car, using volumetric B-Splines as the parameterization tool. Based on industrial experience, an appropriate, though approximate, objective function to be minimized is expressed by the integral of the squared turbulent viscosity over a volume residing next to the driver's window. It should be stressed that if the commonly used "frozen turbulence" assumption was made, by skipping the differentiation of the turbulence model, the adjoint method would not have been able to provide any kind of sensitivity derivatives, since this objective function depends exclusively upon the turbulent viscosity.

1. Introduction

In aerodynamic shape optimization problems, the adjoint method can be used to compute or approximate the gradient of the functions of interest (either objective or constraint functions), with respect to (w.r.t.) the design variables. To this end, the system of adjoint equations must be formulated by taking into consideration the governing PDEs, i.e. the flow model equations and the corresponding boundary conditions. Solving the adjoint system of equations has a computational cost comparable to that of solving the flow equations and is independent of the number of design variables.

The adjoint equations can be formulated in either continuous or discrete form. In the former, the adjoint PDEs and their boundary conditions are derived by processing the objective function augmented by the volume integrals of the state (flow) equations multiplied by the adjoint variables. The adjoint equations are then discretized and solved [1–3]. On the other hand, in the discrete adjoint method the state equations are discretized first and the discrete adjoint equations are deduced from the discretized state equations [4–6]. The present article is concerned exclusively with the continuous adjoint method. Any comparison between the continuous and discrete adjoint methods is beyond the scope of this paper.

When dealing with turbulent flow problems, the state PDEs comprise the mean-flow and turbulence model equations. Computing accurate sensitivity derivatives requires the differentiation of all state PDEs. However, in the majority of articles based on the continuous adjoint method, it is a common practice to avoid the differentiation of turbulence models [2,3,7,8]. This simplification is often referred to as the "frozen-turbulence" assumption. The differentiation of the turbulence model equations using the continuous adjoint method was initially addressed in [9], for the low turbulence Reynolds number variant of the Spalart–Allmaras model for incompressible flows. Later on, the continuous adjoint approach to the same turbulence model for compressible flows was also presented in [10]. The linearization of the turbulence model equations using discrete adjoint is more common and can be found, among other, in [11–15]. A hybrid adjoint method, in which the continuous adjoint mean flow equations are combined with the discrete adjoint turbulence model, was presented in [16].

In [17], the continuous adjoint method was extended to cases in which the mean–flow equations are coupled with the k–ϵ turbulence model and the wall function technique. There, the introduction of the adjoint law of the wall allowed for the adjoint system of PDEs to be treated similarly to the state equations close to the solid walls. In [18], a review on continuous adjoint methods for turbulent flows, including the adjoint to the Spalart–Allmaras model with wall functions was presented, there for objective functions defined

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as surface integrals. Here, the continuous adjoint method to the same model is extended to also cover objective functions defined as volume integrals. The cell-centered, pressure-based implementation of the Spalart–Allmaras model with wall functions as programmed in OpenFOAM© is used as the basis of the developed adjoint formulation. This summarizes the first part of this article (Sections 2–5).

In the second part, the developed adjoint method is used to compute the sensitivity derivatives of an objective function expressing the noise perceived by the driver of a passenger car. According to Proudman's model [19], the acoustic power generated by isotropic turbulence can by derived from Lighthill's Theory of Aerodynamic Noise. Proudman's formula, in turn, can be correlated to the turbulent kinetic energy and dissipation or the levels of turbulent viscosity. So, in order to capture the noise perceived by the driver, the objective function is formulated as the volume integral of the square of the turbulent viscosity over a slim domain residing next to the driver's window. Since the highest levels of turbulence appear at the lower frequencies of the energy spectrum, this is a good surrogate model for low frequency noise. In this case, if the "frozen turbulence" assumption was made, the adjoint method would not have been able to provide any kind of sensitivity information, since the objective depends exclusively on turbulence. Before applying the developed method to a real-world application, its verification is conducted on the flow around an isolated airfoil (Section 7). There, the computed sensitivities are compared to finite differences (FD) and the loss in accuracy caused by avoiding the differentiation of the law of the wall is quantified. Then, the developed adjoint approach is used to optimize the side mirror shape of a passenger car. To this end, volumetric B-Splines [20,21], are used to parameterize the side-mirror surface and handle the mesh displacement during the optimization loop. After a few optimization cycles, a considerable reduction in the objective function is achieved (Section 8).

2. State (Flow) equations

The flow model consists of the Navier–Stokes equations for incompressible flows coupled with the Spalart–Allmaras turbulence model [22]. The mean-flow equations are given by

$$R^p = -\frac{\partial v_j}{\partial x_j} = 0 \tag{1a}$$

$$R_i^v = v_j \frac{\partial v_i}{\partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + \frac{\partial p}{\partial x_i} = 0, \quad i = 1, 2(, 3) \tag{1b}$$

where p is the static pressure divided by the constant density, v_i is the velocity component, $\tau_{ij} = (\nu + \nu_t)(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})$ is the stress tensor, ν is the constant bulk viscosity and ν_t is the turbulent viscosity. In what follows, repeated indices imply summation unless stated otherwise.

The turbulence model equation reads [22],

$$R^{\widetilde{\nu}} = v_j \frac{\partial \widetilde{\nu}}{\partial x_j} - \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\widetilde{\nu}}{\sigma}\right)\frac{\partial \widetilde{\nu}}{\partial x_j}\right] - \frac{c_{b2}}{\sigma}\left(\frac{\partial \widetilde{\nu}}{\partial x_j}\right)^2 - \widetilde{\nu}P + \widetilde{\nu}D = 0 \tag{2}$$

where $\widetilde{\nu}$ is the turbulence state variable. The eddy viscosity coefficient ν_t is expressed in terms of $\widetilde{\nu}$ as follows:

$$\nu_t = \widetilde{\nu} f_{v_1}(\widetilde{\nu}) \tag{3}$$

The model functions and constants can be found in [22] or [9].

Special attention must be paid to the treatment of v_i and ν_t over the boundary faces (index f) where the wall function technique is employed. As programmed in OpenFOAM©, wall functions are based on a single formula modeling both the inner sublayer and the logarithmic part of the turbulent boundary layer [23]

$$f_{WF} = y_P^+ - v_P^+ - e^{-\kappa B}\left[e^{\kappa v_P^+} - 1 - \kappa v_P^+ - \frac{(\kappa v_P^+)^2}{2} - \frac{(\kappa v_P^+)^3}{6}\right] = 0 \tag{4}$$

where $\kappa = 0.41, B = 5.56$ and the non-dimensional distance and velocity at the first cell-center P off the wall are

$$y_P^+ = \frac{\Delta^P v_\tau}{\nu}, \quad v_P^+ = \frac{|v_i|^P}{v_\tau} \tag{5}$$

Here, v_τ is the friction velocity, computed over the wall faces as

$$v_\tau^2 = -\tau_{ij}\big|^f n_j t_i^I \tag{6}$$

where n_j and t_i^I refer to the components of the unit vectors which are normal to the wall and parallel to the velocity (which is considered to be parallel to the wall) at the first cell P. In addition, in 3D simulations, the components of the third unit base vector of a local orthogonal system are defined as $t_i^{II} = e_{ijk} n_j t_k^I$.

Since $v_i^f = 0$, the viscous flux at the wall boundary face f is given by

$$-\tau_{ij}\big|^f n_j = \left(\nu + \nu_t^f\right)\frac{v_i^P}{|Pf|} \tag{7}$$

i.e. the normal velocity gradient at f is computed through a local finite-differences scheme, where $|Pf|$ is the distance of the center of the first cell P to the boundary face. However, the differentiation normal to the boundary must be avoided on the coarse meshes used in conjunction with wall functions. To this end, Eq. (7) is corrected by computing an "artificial" ν_t^f, so that the wall shear stress computed by Eq. (4) and the one computed by differentiating the velocity field and multiplying by ν_t^f be identical. Solving the flow equations using the wall function technique means that Eq. (4) is solved for v_τ, by also taking Eq. (5) into consideration, at each face f using the Newton–Raphson method and, then, Eq. (6) adjusts ν_t^f accordingly.

Finally, the so-called Hamilton–Jacobi equation [24],

$$R^\Delta = \frac{\partial(c_j \Delta)}{\partial x_j} - \Delta \frac{\partial^2 \Delta}{\partial x_j^2} = 0 \tag{8}$$

where $c_j = \partial\Delta/\partial x_j$, is solved to provide the distance to the wall field, Δ, at all interior cell-centers.

3. A general objective function

A general objective function F consisting of a volume integral (such as the one used in Section 6) can be expressed as

$$F = \int_\Omega F_\Omega d\Omega \tag{9}$$

where Ω is the computational domain. The continuous adjoint method, including the adjoint wall functions technique, for objective functions defined as integrals over the boundary S of the computational domain is presented in detail in [18] and will not be discussed herein.

Differentiating F w.r.t. the design variables b_n, $n \in [1, N]$, after taking into consideration the Leibniz theorem for volume integrals with variable boundaries, yields

$$\frac{\delta F}{\delta b_n} = \int_\Omega \frac{\partial F_\Omega}{\partial b_n} d\Omega + \int_S F_\Omega n_k \frac{\delta x_k}{\delta b_n} dS \tag{10}$$

In Eq. (10), the symbol $\delta(\cdot)/\delta b_n$ denotes the total (or material) derivative w.r.t. the design variables and represents the total change caused by variations in b_n. The partial derivative $\partial(\cdot)/\partial b_n$ represents the variation caused purely due to changes in the flow variables (in turn, caused by the geometry deformation) without considering space deformations. The partial and total derivatives of an arbitrary variable Φ are linked through

$$\frac{\delta \Phi}{\delta b_n} = \frac{\partial \Phi}{\partial b_n} + \frac{\partial \Phi}{\partial x_k}\frac{\delta x_k}{\delta b_n} \tag{11}$$