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Lattice-Boltzmann modeling of unstable flows amid arrays of wires

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ABSTRACT

A lattice-Boltzmann model (LBM) of unstable flows amid easily permeable obstructions consisting of regular arrays of wires is presented. The obstacles are modeled by forces incorporated as source terms in the LBM equation, following the same procedure as the immersed boundary method. Yet the present method differs from the latter in that the structure is represented by a volumetric array of fixed points. Also each structural point exerts reactive forces governed by the Darcy law, rather than by elastic kinematics. The model is validated against two experiments consisting of air flowing in a channel partially obstructed by arrays of wires, finding excellent agreement. The simulations reveal the formation of complex vortical structures amid the wired region, which can be of interest in understanding natural phenomena or practical applications.

1. Introduction

The numerical modeling of flows in easily permeable media, like the flow amid complex arrays of small obstacles, is a challenging task. This kind of flows occurs in a number of interesting cases, like fluid currents around crops or aquatic plants [6,13,21]. In technological applications they are encountered for example in the cooling of electronic components. A comprehensive review on compound channels and flows parallel to rod bundles can be found in Meyer [15].

An interesting feature of these flows is that they easily become unstable in the boundaries between the permeable regions and the free flow [28]. This instability is associated to inflexion points of the velocity profile, and manifests as coherent wavy and vortical structures that amplify the momentum and scalar transversal diffusion [3,18]. These structures are useful for they modify the transport of scalars not only at the interface where structures are generated, but also inside and outside the obstruction, favoring processes such as pollination, nutrient transport, and heat and mass transfer [5,23]. A number of recent analytical studies have been aimed to characterize the stability of partially permeable channels [14,24,25,28].

From the point of view of modeling, finding adequate representations of the arrays of obstacles, that capture the characteristics of the mentioned instabilities while keeping reasonable computational costs, is not a trivial task. The lattice Boltzmann method (LBM) has shown an impressive versatility to model porous media and flows through regions partially blocked by obstacles ([19,7,10,27]). For a good review of applications of LBM to porous media (see Sukop and Thorne [26]). Moreover, recently it has been shown that flow instabilities in cavities can be simulated using LBM [2,8].

The present article reports the modeling of the sustained oscillatory flow in a channel with low Reynolds number partially obstructed by an array of wires. The wired region is modeled by Darcy-like forces imposed around the location of each wire. This is a major difference from a previous approach based on local bounce-back corrections, which produced less accurate results and introduced spurious mass sources [4]. The model is validated against two experiments consisting of air flowing in a channel partially obstructed by arrays of wires in different configurations.

2. Modeling

From the numerical point of view, LBM can be seen as an explicit method to solve transport equations using more variables than the strictly necessary to characterize the macroscopic flow. It is based on the movement and collision of pseudo-particles described by the lattice–Boltzmann equation:

$$f_i(\vec{x}+\vec{e}_i\Delta x, t+\Delta t) = f_i(\vec{x},t) - \frac{1}{\tau}\left[f_i(\vec{x},t) - f_i^e(\vec{x},t)\right] + S_i(\vec{x},t),\ \text{for } i = 0,\ldots,\ell-1 \quad (1)$$

where $f_i(\vec{x},t)$ and $S_i(\vec{x},t)$ represent the particles distribution density and source at position $\vec{x}$ and time t, undergoing a displacement $\vec{e}_i\Delta x$ in a time step Δt. The vectors $\vec{e}_i$ form a finite set of ℓ lattice directions

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that restrict the movement of the particles. In what follows the so called D2Q9 model will be used, which approach to second order the two-dimensional Navier–Stokes equations and is given by the set:

$$\vec{e}_0 = (0,0);\ \vec{e}_1 = (1,0);\ \vec{e}_2 = (0,1);\ \vec{e}_3 = (-1,0);\ \vec{e}_4 = (0,-1);\ \vec{e}_5 = (1,1);\ \vec{e}_6 = (-1,1);\ \vec{e}_7 = (-1,-1);\ \vec{e}_8 = (1,-1). \quad (2)$$

Using an asymptotic expansion, it has been demonstrated that the LBE approximates the Navier–Stokes equations, provided that the so called equilibrium function $f_i^e(\vec{x},t)$ satisfies a set of constitutive conditions related to the moments of $f_i(\vec{x},t)$ respect to $\vec{e}_i$. Comprehensive reviews of this procedure can be found elsewhere [9,22,29]. A popular scheme complying with these conditions is the classical BGK, which for D2Q9 is given by:

$$f_i^e(\vec{x},t) = w_i\rho\left[1 + 3\frac{(\nu\vec{e}_i\cdot\vec{u})}{\nu^2} - \frac{3}{2}\frac{u^2}{\nu^2} + \frac{9}{2}\frac{(\nu\vec{e}_i\cdot\vec{u})^2}{\nu^4}\right], \quad (3)$$

where $\nu = \Delta x/\Delta t$ is the particle speed and:

$$\rho = \sum_i f_i(\vec{x},t) \text{ and } \vec{u} = \frac{1}{\rho}\sum_i \nu\vec{e}_i f_i(\vec{x},t) \quad (4)$$

are the particle-number density and average velocity. The coefficients w_i are 4/9 for the resting particles, 1/9 for the Cartesian directions and 1/36 for the diagonal directions. In such case, the relaxation parameter τ is related to the kinematic viscosity of the fluid by:

$$\upsilon = (2\tau - 1)\ \Delta x^2/(6\Delta t) \quad (5)$$

and the pressure is calculated using the isothermal pseudo equation of state:

$$p = \frac{1}{3}\rho\nu^2. \quad (6)$$

The term $S_i(\vec{x},t)$ is generally used to account for external forces. There is certain flexibility to manage this term. In the present work, it is assumed that a volumetric force $\vec{F}(\vec{x},t)$ can be applied in each cell, and the following expression recommended by Mohamad & Kuzmin [16] will be used:

$$S_i(\vec{x},t) = 3\frac{\Delta t}{\nu}w_i\vec{e}_i\cdot\vec{F}(\vec{x},t). \quad (7)$$

In effect, contracting the velocity index of S_i and $\vec{e}_iS_i$ yields:

$$\sum_i S_i = 3\frac{\Delta t}{\nu}\sum_i w_i\vec{e}_i\cdot\vec{F} = 0 \quad (8)$$

which ensures mass conservation, and

$$\sum_i \nu\vec{e}_iS_i = 3\ \Delta t\ \sum_i w_i\ \vec{e}_i\otimes\vec{e}_i\cdot\vec{F} = 3\Delta t\begin{pmatrix}1/3 & 0\\ 0 & 1/3\end{pmatrix}\cdot\vec{F} = \Delta t\vec{F} \quad (9)$$

provides the momentum bit to the cell driven by the volumetric force.

2.1. Model of the permeable media with LMB

The permeable media was simulated by means of source terms accounting for the drag forces imposed by the array of obstacles in the channel. The rationale behind this method is to simulate the Darcy's law, which states that the volumetric drag force $\vec{F}_D$ of permeable media is given by:

$$\vec{F}_D = -\frac{\rho\upsilon}{\kappa}\vec{u} \quad (10)$$

where κ is the permeability of the media.

The direct way to incorporate Darcy's law in LBM is to distribute the force uniformly in each cell of the permeable region, by introducing Eq. 10 in the source term. Although this is attractive for its simplicity, the numerical tests showed significant differences with the experimental data, particularly in transient and unstable conditions. In the present case, the main reason of the poor performance of a homogeneous model of forces is that the actual permeable media where the experiments were carried out is not homogeneous but it is composed by discrete thin wires that oppose to the flow locally. In order to simulate more closely the wire array while maintaining the simplicity of Darcy's law, another model is proposed in which the force term is applied in the neighborhood of each wire location. This is implemented following the same procedure as in the immersed-boundary method (IBM) used in fluid-structure interaction schemes [20,1].

In IBM the fluid is represented on an Eulerian coordinate whereas structures are represented by collections of parametric curves or surfaces on a Lagrangian coordinate. The forces exerted by the immersed boundary on the fluid are incorporated as source terms in the fluid equation via smooth approximations of the Dirac δ distribution. The immersed boundary is considered as a massless elastic fiber or membrane that moves with the local fluid velocity interpolated with the same δ distribution approximation.

In the present approach permeable structures are coupled with the fluid through smooth approximations of the δ distribution as in IBM. However, the present method differs from IBM in that the structure is represented by volumetric arrays of fixed points on the same Eulerian coordinate grid as the fluid, over which each structural point exerts reactive forces governed by the Darcy law rather than by elastic kinematics. The permeable region is accordingly defined by a set of reference points x_k, each of which introduces a source term $S_{ki}(\vec{x},t)$ to the cells located in its neighborhood, given by:

$$S_{ki}(\vec{x},t) = -3\frac{\Delta t}{\nu}\frac{\rho\upsilon}{\kappa}N_k\delta(\vec{x}_k,\vec{x})\ w_i\ \vec{e}_i\cdot\vec{u}_k \quad (11)$$

where N_k is the number of LB cells that constitute a unit cell of the permeable media and

$$\delta(\vec{x}_k,\vec{x}) = \begin{cases} C(r)\left[1+\cos\frac{\pi(x-x_k)}{r\,\Delta x}\right]\left[1+\cos\frac{\pi(y-y_k)}{r\,\Delta x}\right] & if \left|\frac{(x-x_k)}{r\,\Delta x}\right| and \left|\frac{(y-y_k)}{r\,\Delta x}\right| < 1 \\ 0 & \text{otherwise}\end{cases}. \quad (12)$$

In other words, the force is distributed among the cells located at a distance $r\Delta x$ in each direction from the reference point. The radius of influence r is used for calibration purposes and it is expected to depend on the obstacle specifics (shape, size, roughness, etc.). $C(r)$ is a normalization factor given by:

$$C(r) = \left[\sum_{n=-r}^{r}\left(1+\cos\frac{\pi n}{2r}\right)\right]^{-2}. \quad (13)$$

The characteristic velocity of the neighborhood $\vec{u}_k$ is defined as:

$$\vec{u}_k = \sum_{\vec{x}}\delta(\vec{x}_k,\vec{x})\ \vec{u}(\vec{x}). \quad (14)$$

In Eq. (14) the summation is performed over all the cells of the grid, although the factor $\delta(\vec{x}_k,\vec{x})$ restricts the effect only to the cells within the zone of influence of the reference point $\vec{x}_k$.

3. Experiment

An experimental setup was constructed in order to provide a reference case and reliable measurements to compare with the numerical model. The test section, shown in Fig. 1, is a rectangular channel with transparent acrylic walls. The working fluid is ambient air that enters the channel from one end and is forced out by a fan located on the other end. The obstructed region consists of a regular array of copper wires lined horizontally across the test section, occupying