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Large-eddy simulation with near-wall modeling using weakly enforced no-slip boundary conditions

Roozbeh Golshan [a], Andrés E. Tejada-Martínez [a,*], Mario Juha [a], Yuri Bazilevs [b]

[a] Department of Civil & Environmental Engineering, University of South Florida, 4202 East Fowler Avenue ENB 118, Tampa, FL 33620, USA
[b] Department of Structural Engineering, University of California, San Diego, 9500 Gilman Drive MC0085, La Jolla, CA 92093, USA

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ABSTRACT

In the present paper, weakly enforced no-slip wall boundary conditions are revisited in the context of Large-Eddy Simulations (LES) with near-wall modeling. A new formulation is proposed in the framework of weakly enforced no-slip conditions that is better aligned with traditional near-wall modeling approaches than its predecessors. The new formulation is tested on turbulent open-channel flows at friction-velocity-based Reynolds numbers $Re_\tau = 395$ and 950 benchmark problems. The computations are performed using the Residual-Based Variational Multiscale (RBVMS) formulation of LES, discretized using Isogeometric Analysis (IGA) based on Non-Uniform Rational B-Splines (NURBS). The new near-wall model formulation gives more accurate results for the mean flow and velocity fluctuations than its older versions, while exhibiting better numerical stability than traditional near-wall modeling techniques.

1. Introduction

In large-eddy simulations (LES) [31] of wall-bounded turbulent flows, near-wall sharp gradients of the velocity field, and small eddies scaling with distance to the wall, give rise to undesired computational costs associated with fine grids and small time steps needed to resolve these features. Near-wall modeling is often employed to significantly reduce these costs [28]. In LES with near-wall modeling (LES-NWM), while the core flow is reasonably well resolved, the unresolved near-wall region is modeled through suitable boundary conditions, which obviates the need to use small time steps and fine meshes in the near-wall region. This is in contrast to LES with near-wall resolution (LES-NWR), in which the mesh is made finer near the wall, and the time-step size is reduced in order to capture the finer spatial and temporal scales of near-wall turbulence. With the significant reduction in computational cost brought about with near-wall modeling, more realistic problems in terms of computational-domain size, geometric complexity, and Reynolds number may be solved with LES-NWM as opposed to with LES-NWR or direct numerical simulation (DNS). The latter two are mostly employed to study turbulent-flow physics at relatively low Reynolds number in configurations of reduced geometric complexity.

In traditional near-wall modeling, instead of imposing the no-slip boundary conditions at the wall, the wall shear stress boundary conditions are prescribed. The following procedure is usually employed: Assuming the LES-resolved mean flow satisfies the log law, which is part of the well-known law-of-the-wall [32], the associated wall friction velocity is extracted, and, in turn, used to compute the magnitude of the prescribed wall shear stress. (The direction of the prescribed wall shear stress vector is given by the slip velocity.) Because the wall friction velocity is a nonlinear function of the LES-resolved flow velocity, an iterative procedure is necessary to compute it. We also note that the LES-resolved mean velocity used to extract the wall shear stress from the law-of-the-wall is often computed by averaging the flow field in space, typically over homogeneous flow directions, which makes the stress boundary conditions nonlocal.

An alternative approach to reduce mesh- and time-step-size requirements in wall-bounded turbulent flows was introduced in [8]. The approach is based on so-called weak enforcement of Dirichlet no-slip conditions at the solid wall. More specifically, the formulation at the discrete level is based on the variational (or weak) form of the Navier–Stokes equations, which is augmented by terms that enforce no-slip conditions weakly as Euler–Lagrange conditions. This construction results in a numerical technique that satisfies no-slip conditions exactly only in the limit as the mesh size at the wall approaches zero. As a result, for a given mesh, the method allows a certain amount of flow slip at the wall,

* Corresponding author. Tel.: +1 813 974 1738; fax: +1 813 974 2275.
E-mail address: aetejada@usf.edu (A.E. Tejada-Martínez).

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which scales with the appropriate power of the wall-normal mesh size. Allowing the flow to slip at the solid boundary removes some of the burden from the boundary-layer mesh to resolve the sharp velocity gradients near the wall, which, in turn, results in remarkably accurate solutions for boundary-layer flows on meshes that would be considered "too coarse" by most CFD practitioners. (See Refs. [10,9,1,18,19] that highlight the coarse-mesh accuracy of weakly enforced no-slip conditions, both for well-known turbulent flow benchmark problems as well as for engineering applications.)

The method of weak enforcement of no-slip conditions is an extension of Nitsche's idea for imposing essential boundary conditions weakly [27], and is now commonly referred to as Nitsche's method. The method may also be thought of as the SIPG Discontinuous Galerkin technique (see [40]) applied only at the solid boundary. An important component of the weak boundary condition formulation is the penalty term whose integrand contains the deviation of the discrete solution from the no-slip condition at the wall. The integrand of the penalty term is also proportional to a mesh-dependent penalty parameter τ_B designed to ensure numerical stability and optimal convergence under mesh refinement. As such, weak imposition of no-slip conditions is based on numerical considerations rather than physical ones.

The link between weakly enforced no-slip conditions and near-wall modeling was first recognized and exploited in [10]. In the aforementioned reference the parameter τ_B was computed following the law-of-the-wall by considering the penalty term as being representative of the shear stress at the wall. While slight improvement in the results was observed relative to the purely numerical design of τ_B, we recognize that the formulation in [10] was not completely consistent with the law-of-the-wall for the following reasons: 1. Only the penalty term was representative of the shear stress at the wall, while the resolved molecular viscous shear stress was neglected; 2. The resolved LES velocity at the wall, instead of that inside the log layer, was employed to compute the wall shear stress and the penalty parameter τ_B. This motivates the development of an improved formulation for weak enforcement of the non-slip boundary conditions, which we present in this paper. The merit of this new formulation is that it is designed to inherit the positive numerical attributes of the original formulations (i.e., stability and optimal convergence) while being consistent with the-law-of-the-wall. Furthermore, this better alignment with the law-of-the-wall opens the door for future improvements of the new formulation following the developments already made for traditional near-wall modeling [28].

The manuscript is organized as follows. Section 2 presents the Navier–Stokes equations of incompressible flows in the strong and weak forms and provides some details of the residual-based variational multiscale (RBVMS) formulation of LES [6,2,11]. Section 3 provides the details of a traditional near-wall model, as well as near-wall models based on weak imposition of no-slip conditions. The new near-wall model formulation is presented in Section 4. Sections 5 focuses on the numerical results. We compute pressure-gradient-driven turbulent open-channel flows at friction-velocity Reynolds numbers $Re_\tau = 395$ and 950 using an isogeometric analysis (IGA) discretization based on Non-Uniform Rational B-Splines (NURBS) [22,15]. A comparison of LES-NWM results for the traditional near-wall model, weak BC formulation from [10,9], and its improved version proposed in the current work is performed. Conclusions are drawn in Section 6.

2. Navier–Stokes equations, RBVMS formulation, and near-wall modeling

Let $\Omega \in \mathbb{R}^3$ be the problem domain and let Γ denote its boundary. A conservative form of the dimensionless Navier–Stokes equations of incompressible flows in the Eulerian frame may be written as

$$\frac{\partial \boldsymbol{u}}{\partial t} + \nabla\cdot(\boldsymbol{u}\otimes\boldsymbol{u}) + \nabla p - \nabla\cdot(2\nu\nabla^s\boldsymbol{u}) = \boldsymbol{f} \text{ in } \Omega, \tag{1}$$

$$\nabla\cdot\boldsymbol{u} = 0 \text{ in } \Omega, \tag{2}$$

where Eqs. (1) and (2) represent conservation of linear momentum and mass, respectively, assuming the fluid density is constant. In the above equations, $\boldsymbol{u}$ and p are the fluid velocity and pressure (divided by density), ν is the kinematic viscosity, $\nabla^s = \frac{1}{2}\left(\nabla + (\nabla)^T\right)$ is the symmetric spatial gradient, and $\boldsymbol{f}$ is a body force per unit mass.

The RBVMS formulation of the Navier–Stokes equations of incompressible flows is employed in this work [6,2,11]. RBVMS originates from stabilized and multiscale methods for fluid mechanics [13,39,21,23]. In [6] it was derived and presented for the first time in the context of subgrid-scale modeling for LES. In [5] a moving-domain extension of RBVMS was presented in the framework of an Arbitrary Lagrangian–Eulerian (ALE) formulation [20], and later called the ALE-VMS method [33]. The space–time version of RBVMS, called ST-VMS, was recently proposed in [36] and successfully employed in a number of fluid mechanics and fluid–structure interaction simulations in [33,7,35,37,34]. In [6] it was shown RBVMS performs well on laminar and turbulent flows, and discrete solutions converge rapidly to DNS while yielding LES-like solutions on intermediate meshes. For better approximation of thin boundary layers near no-slip walls various wall-modeling approaches may be used. Here we focus on classical wall-function-based techniques as well as more recently proposed methods for weak enforcement of the no-slip Dirichlet boundary conditions [8].

The space-discrete formulation that combines RBVMS and near-wall modeling may be stated as: Find $\{\boldsymbol{u}^h, p^h\} \in \mathcal{V}^h$ such that $\forall\{\delta\boldsymbol{u}^h, \delta p^h\} \in \mathcal{W}^h$,

$$B(\{\delta\boldsymbol{u}^h,\delta p^h\},\{\boldsymbol{u}^h,p^h\}) + B_{vms}(\{\delta\boldsymbol{u}^h,\delta p^h\},\{\boldsymbol{u}^h,p^h\}) + B_{wm}(\{\delta\boldsymbol{u}^h,\delta p^h\},\{\boldsymbol{u}^h,p^h\}) = (\delta\boldsymbol{u}^h,\boldsymbol{f})_\Omega, \tag{3}$$

where $\mathcal{V}^h$ denotes the discrete solution space for velocity–pressure pairs $\{\boldsymbol{u}^h, p^h\}$, $\mathcal{W}^h$ denotes the discrete space of linear-momentum and continuity-equation test-function pairs $\{\delta\boldsymbol{u}^h, \delta p^h\}$, and $(\cdot,\cdot)_A$ denotes an L_2-inner product over the domain A.

The semi-linear forms in the formulation given by Eq. (3) are defined in what follows.

$$B(\{\boldsymbol{w},q\},\{\boldsymbol{u},p\}) = \left(\boldsymbol{w},\frac{\partial\boldsymbol{u}}{\partial t}\right)_\Omega - (\nabla\boldsymbol{w},\boldsymbol{u}\otimes\boldsymbol{u})_\Omega + (q,\nabla\cdot\boldsymbol{u})_\Omega - (\nabla\cdot\boldsymbol{w},p)_\Omega + (\nabla^s\boldsymbol{w},2\nu\nabla^s\boldsymbol{u})_\Omega, \tag{4}$$

is the Galerkin part of the weak form. Furthermore,

$$B_{vms}(\{\boldsymbol{w},q\},\{\boldsymbol{u},p\}) = -(\nabla\boldsymbol{w},\boldsymbol{u}'\otimes\boldsymbol{u} + \boldsymbol{u}\otimes\boldsymbol{u}' + \boldsymbol{u}'\otimes\boldsymbol{u}')_\Omega - (\nabla\cdot\boldsymbol{w},p')_\Omega - (\nabla q,\boldsymbol{u}')_\Omega, \tag{5}$$

are the RBVMS terms, where the pair $\{\boldsymbol{u}', p'\}$ denotes the velocity and pressure subgrid scales (i.e., the scales that are too small to be reasonably approximated on a given mesh). As in [6], the subgrid scales are modeled as

$$\boldsymbol{u}' = -\tau_M\left(\frac{\partial\boldsymbol{u}}{\partial t} + \boldsymbol{u}\nabla\boldsymbol{u} + \nabla p - \nu\Delta\boldsymbol{u}, -\boldsymbol{f}\right)$$
$$p' = -\tau_C\nabla\cdot\boldsymbol{u}, \tag{6}$$

where τ_M and τ_C are the subgrid-scale parameters defined later in the section. The subgrid-scale parameters are also known as stabilization parameters due to the similarities between RBVMS and