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Free convection in a porous horizontal cylindrical annulus with a nanofluid using Buongiorno's model

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ABSTRACT

Natural convection flow in a porous concentric horizontal annulus saturated with a water based nanofluid is numerically investigated. The mathematical model used is of single-phase and is formulated in dimensionless stream function and temperature taking into account the Darcy–Boussinesq approximation and the nanofluid model proposed by Buongiorno. The transformed dimensionless partial differential equations have been solved using a second-order accurate finite-difference technique. The results indicate that inclusion of nanoparticles into pure water changes the flow structure at low values of the Rayleigh number.

1. Introduction

Porous media appear everywhere. Examples range from civil, chemical and geological engineering until an essential part of our daily lives to live comfortably [1]. A few examples of porous media are: soils, aquifers, sands, clothing, filters and catalytic converters in cars, ground water flow, etc. Convective flow and heat transfer is, generally, prevalent in fields of physics and engineering, such as geothermal reservoirs, float glass production, flow and heat transfer in solar ponds, air-conditioning in rooms and cooling of electronic devices [2]. Natural or free convection in a porous cavity has received considerable attention in recent years because of its relation to the thermal performance of many engineering installations [3]. The phenomenon of natural convection in porous media is a fundamental transport mechanism encountered in a wide range of engineering, geophysics, and scientific applications, such as packed bed solar energy storage, fibrous and granular insulation systems, water reservoirs and post-accident cooling of nuclear reactors [4]. Despite the fact that buoyant convection in this system was first studied about 40 years ago, there has lately been renewed interest in such flow in porous cavities owing to its importance in environmental and energy management problems in current scientific and geo-political context [4]. Application and control concept of flow through porous media to oil and gas reservoir simulation, geothermal energy, or groundwater remediation are important research topics in porous media nowadays [5].

As traditional fluids used for heat transfer applications such as water, mineral oils and ethylene glycol have a rather low thermal conductivity, nanofluids with relatively higher thermal conductivities have attracted enormous interest from researchers due to their potential in enhancement of heat transfer with little or no penalty in pressure drop [6]. It seems that Choi [7] is the first who introduced the term nanofluid to describe the mixture of nanoparticles and base fluid such as water, mineral oils and ethylene glycol. The addition of nanoparticles into the base fluid is able to change the flow and heat transfer capability of the liquids and indirectly increase the low thermal conductivity of the base fluid which is identified as the main obstacle in heat transfer performance. This mixture has attracted the interest of numerous researchers because of its many significant applications such as, for example, in the medical applications, transportations, microelectronics, chemical engineering, aerospace and manufacturing [8]. The convective heat transfer characteristic of nanofluids depends on the thermo-physical properties of the base fluid and the ultra fine particles, the flow pattern and flow structure, the volume fraction of the suspended particles, the dimensions and the shape of these particles. The utility of a particular nanofluid for a

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heat transfer application can be established by suitably modeling the convective transport in the nanofluid [6].

Many authors such as Kakaç and Pramuanjaroenkij [9], Wong and Leon [10], Fan and Wang [11], Jaluria et al. [12], Mahian et al. [13], Hajmohammadi et al. [14], Soleimani et al. [15], Ashorynejad et al. [16] and Seyyedi et al. [17] have presented comprehensive literature review on nanofluids. For example, Hajmohammadi et al. [14] analyzed nanofluid flow and heat transfer over a permeable flat plate using convective boundary condition. It was shown that the increase in skin friction is a considerable drawback imposed by Cu/water and Ag/water nanofluids, especially in the case of injection. Soleimani et al. [15] investigated numerically natural convection inside the semi-annulus cavity filled with a nanofluid. They found that the effect of the nanoparticles is more pronounced at low Rayleigh number than at high Rayleigh number because of greater amount of enhancement and increasing Rayleigh number leads to a decrease in ratio of heat transfer enhancement. Ashorynejad et al. [16] studied numerically on the basis of the Lattice Boltzmann method the effect of magnetic field on natural convection in a horizontal cylindrical annulus filled with a nanofluid. It has been shown that the average Nusselt number is an increasing function of nanoparticle volume fraction and Rayleigh number, while it is a decreasing function of Hartmann number. Seyyedi et al. [17] using the finite volume method numerically analyzed the natural convective heat transfer in an annulus filled with a Cu/water nanofluid. It has been found that the angle of turn for the boundary condition of the inner cylinder essentially affects the average Nusselt number. However, these papers are based on the mathematical nanofluid models proposed by Khanafer [18], and Tiwari and Das [19] for the two-phase mixture containing micro-sized particles. On the other hand, one should also mention the mathematical nanofluid model proposed by Buongiorno [20] used in many papers pioneered by Kuznetsov and Nield [21] for the free convection boundary layer flow along a vertical flat plate embedded in a porous medium and, Nield and Kuznetsov [22] for the problem of thermal instability in a porous medium layer saturated by a nanofluid. In this model, the Brownian motion and thermophoresis enter to produce their effects directly into the equations expressing the conservation of energy and nanoparticles, so that the temperature and the particle density are coupled in a particular way, and that results in the thermal and concentration buoyancy effects being coupled in the same way. We also mention here the very recently published review paper by Sakai et al. [23], where a macroscopic set of the governing equations for describing heat transfer in nanofluid saturated porous media were rigorously derived using a volume averaging theory, for possible heat transfer applications of metal foams filled with nanofluids as high performance heat exchangers. Metal foams filled with nanofluids can be one of the most promising candidates for a high performance heat exchanger needed for highly concentrated heat generating devices [24].

Literatures indicate that convection heat transfer inside concentric and eccentric annuli has many applications in science and engineering, such as electrical motor and generator, completion of an oil source and heating and cooling of underground electric cables. Recently, investigation of the effect of eccentricity on heat transfer has become a subject of interest to most researchers working in the area of convection heat transfer problems. It seems that the first person who worked on eccentric annuli was Heyda [25], and he applied a fundamental solution known as Green's function on solving the momentum equation for a laminar flow inside an eccentric annulus. A valuable list of references on this problem can be found in the paper by Matin and Pop [26]. It is, however, worth mentioning that El-Amin et al. [27] have founded a mathematical model of nanoparticles transport in two-phase flow in porous media based on the formulation of fine particles transport in two-phase flow in porous media proposed by Liu and Civian [28].

The principal objective of the present paper is to analyze the steady natural convection in a porous horizontal cylindrical annulus with a nanofluid using the single-phase mathematical nanofluid model proposed by Buongiorno [20]. To our best of knowledge this problem has not been considered before, so that the reported results are new and original.

It is worth noting that in the present paper we use the numerical method for solution to the boundary value problem for the partial differential equations. At the same time it is possible to transform these equations to ODE's and to solve by the semi-analytical methods [29–33]. Such algorithms [30,34,35] can allow to study the stability of the convective flows.

2. Basic equations

Consider the steady free convection flow and heat transfer in a porous horizontal annulus filled with a water based nanofluid. It is assumed that nanoparticles are suspended in the nanofluid using either surfactant or surface charge technology. This prevents nanoparticles from agglomeration and deposition on the porous matrix (see Kuznetsov and Nield [21]; Nield and Kuznetsov [22]). A schematic geometry of the problem under investigation is shown in Fig. 1, where $\bar{r}$ and γ are the polar system of coordinates and $\bar{r}_2 - \bar{r}_1$ is the size of the annulus. It is assumed that the internal surface $\bar{r} = \bar{r}_1$ is heated and maintained at the constant temperature T_h, while the external surface $\bar{r} = \bar{r}_2$ is cooled and has the constant temperature T_c. If we assume that the angular coordinate is measured clockwise from the vertically down position and that the problem is symmetric about a vertical plane passing through the axis of the cylinder, then consideration will be confined to the range $0 < \gamma < \pi$ (or $\pi < \gamma < 2\pi$).

The basic equations for the flow, heat transfer and nanoparticles can be written in the following form (see Buongiorno [20]; Kuznetsov and Nield [21]; Nield and Kuznetsov [22]),

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$0 = -\nabla p - \frac{\mu}{K}\mathbf{V} + \left[C\rho_p + (1-C)\rho_{f0}(1-\beta(T-T_c))\right]\mathbf{g} \tag{2}$$

$$\sigma\frac{\partial T}{\partial t} + (\mathbf{V}\cdot\nabla)T = \alpha_m\nabla^2 T + \delta[D_B\nabla C\cdot\nabla T + (D_T/T_c)\nabla T\cdot\nabla T] \tag{3}$$

$$\rho_p\left(\frac{\partial C}{\partial t} + \frac{1}{\varepsilon}(\mathbf{V}\cdot\nabla)C\right) = -\nabla\cdot\mathbf{j}_p \tag{4}$$

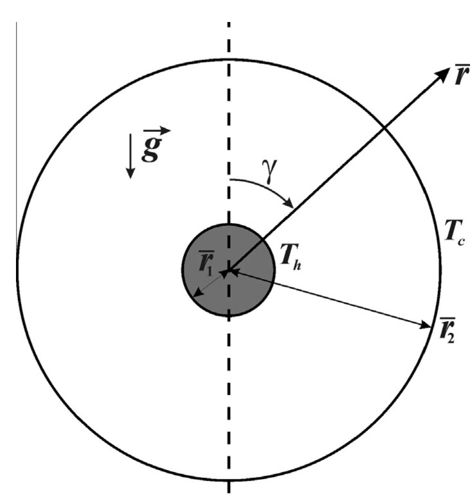

Fig. 1. Physical model and coordinate system.