Computers & Fluids 115 (2015) 115–123

Contents lists available at ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

Implicit dual-time stepping method for a solar wind model in spherical coordinates

Man Zhang, Xueshang Feng *

SIGMA Weather Group, State Key Laboratory for Space Weather, Center for Space Science and Applied Research, Chinese Academy of Sciences, Beijing 100190, China

ARTICLE INFO

Article history:
Received 4 September 2014
Received in revised form 8 March 2015
Accepted 26 March 2015
Available online 3 April 2015

Keywords:
Implicit dual time-stepping method
Magnetohydrodynamics equations
Solar wind

ABSTRACT

In this paper, an implicit dual-time stepping scheme based on the finite volume method in spherical coordinates with a six-component grid system is developed to model the steady state solar wind. By adding a pseudo-time derivative to the magnetohydrodynamics equations for the solar wind plasma, the governing equations are solved implicitly at each physical time step by advancing in pseudo time. As a validation, ambient solar wind for Carrington rotation 2048 has been studied. Numerical tests with different Courant factors show its capability of producing structured solar wind and that the physical time step can be enlarged to be one hundred times that of the original one. Our numerical results have demonstrated overall good agreements with the observations.

1. Introduction

Over the last few decades, numerical simulations of the solar wind plasma flow have evolved from a topic only addressed in basic research toward a promising tool used for space weather prediction. Today's maturation of computational magnetohydrodynamic (MHD) dynamics has enabled us to numerically capture the basic structures of the solar wind plasma flow and transient phenomena such as the solar wind background and coronal mass ejections (CMEs) [1]. Of course, besides the numerical aspect, numerical space weather modeling depends heavily on the understanding of the fundamental physics processes, such as the coronal heating/solar wind acceleration, initiation of solar eruptions like CMEs, which will benefit from further theoretical investigation, and spacecraft observation. This rapid development of numerical space weather modeling can be attributed to both the achievement of efficient solution algorithms and the continuous increase in available computational power. With today's level of maturity in numerical algorithms, it is tempting to assume that further progress in the applicability of numerical methods may be guaranteed by solely relying on the sustained development of computer technology. However, since the relevant problem size will continue to increase as fast as available hardware permits, a number of severe challenges in the development of numerical methods for solar wind plasma flow simulation remain. Globally, without going into detail, these challenges may be summarized by the terms efficiency, robustness, and accuracy, as usually met in other computational fields [2,3].

The MHD description governs the large-scale dynamics of solar wind plasmas. Mathematically, ideal MHDs form a hyperbolic partial differential equation (PDE) system, in which seven waves appear, labeled as entropy, forward and backward slow, forward and backward Alfvén, and forward and backward fast families, which all behave anisotropic. The associated seven wave speeds are the local velocity, v, and the sets are $v \pm v_{\mathrm{Slow}}, v \pm v_{\mathrm{Alfven}}$, $v \pm v_{\mathrm{Fast}}$, where v_{Slow} denotes slow magnetoacoustic speeds, while v_{Fast} indicates fast. Together with the Alfvén speed, v_{Alfven}, they are ordered since $v_{\mathrm{Slow}} \leqslant v_{\mathrm{Alfven}} \leqslant v_{\mathrm{Fast}}$. Currently, many solar-terrestrial physics phenomena that require the solution of a hyperbolic system of MHD equations involve vastly different physical timescales and spatial scales. With respect to efficiency in the numerical modeling of solar-terrestrial physics phenomena, one of the major breakthroughs in numerical methods for MHD simulations was the introduction of adaptive mesh refinement [4–7], and for the solution of the inviscid equations, numerical methods may now be considered as fairly effective. However, we are still frustrated by the inadequacy of today's methods to efficiently take into account the stiffness of the discrete system of equations. For a stiffness discussion in fluid dynamics, we can refer to [2,8] and references therein. For MHD equations, the same arguments hold. Discrete stiffness can generally result from distinct sources due to the use of a scalar time step, highly stretched computational meshes, and/or other physics such as dissipative/heating processes in the form of source terms. The scalar time step can fail to cope with the disparity in the propagation speeds of convective and

* Corresponding author.
E-mail address: fengx@spaceweather.ac.cn (X. Feng).

http://dx.doi.org/10.1016/j.compfluid.2015.03.020
0045-7930/

characteristic wave modes and the highly stretched computational meshes can be required for economical resolution of the spherical shell computational domain from solar corona to interplanetary space. The discrete stiffness, provoked by the highly stretched computational meshes, can be enhanced with the increase of the corresponding high cell aspect ratios by several orders of magnitude in large portions of the computational domain which results in severe convergence problems and very high computation times, particularly in solar wind simulation studies [3].

In general, the MHD equations of the solar wind plasma involve a wide separation in timescales. The slow wave and Alfvén wave only propagate in the direction parallel to the background magnetic field whereas the propagation of the fast wave is nearly isotropic [9]. Hence, the spatial resolution requirements perpendicular to the magnetic field are much more severe than those parallel to the magnetic field, making the Courant–Friedrichs–Lewy (CFL) condition associated with the fast wave much more restrictive than that associated with the others; typically by two or more orders of magnitude. Since the fast wave is the only one that compresses the magnetic field, the fast wave sets the maximum allowable time step when using an explicit time advance. It is generally believed that when the MHD equations are used to study plasma phenomena occurring on time scales as short as the transit time of a fast MHD wave, an implicit scheme removes the numerically imposed time-step constraint allowing much larger time steps [9,10]. Besides implicit time integration, the use of dual time stepping, allows, to some extent, the physical time step to not be limited by the corresponding values in the smallest cell and to be selected based on the numerical accuracy criterion [2]. The dual time step, which does not modify the original transient evolution of the governing equation, adds a pseudo-time derivative to the governing equation. It uses the pseudo-time steady-state solution to approach the physical-time solution. A dual time marching method for MHD-like equations has been used [11–15] for the simulation of MHD phenomena.

The objective of the present paper is to explore an implicit dual time-stepping method for 3D MHD studies of ambient solar wind.The paper is organized as follows. In Section 2, the governing MHD equations of the solar wind plasma in spherical coordinates are briefly provided. In Section 3, the hybrid finite volume scheme of combining the fluid part of the MHD equations and the constraint transport method for the magnetic induction part with dual time stepping are described. In Section 4, the numerical results of ambient solar wind for Carrington rotation (CR) 2048 with different CFL numbers or Courant factors are presented. Finally, conclusions are made.

2. Governing equations

The magnetic field, $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_0$, is split into the sum of a time-independent potential magnetic field, B_0, and a time-dependent deviation, B_1 [16,17]. Here, $\mathbf{B_0}$ is a potential magnetic field, and in the present paper it is taken as the initial value with $\frac{\partial \mathbf{B_0}}{\partial t} = 0, \nabla \cdot \mathbf{B}_0 = 0, \nabla \times \mathbf{B}_0 = 0$. We note that MHD equations can be viewed as a combination of fluid dynamics coupled with magnetic fields. In the present paper, this physical splitting of the MHD equations into fluid and magnetic parts [18,19] is considered in order to design efficient finite volume (FV) schemes with spatial discretization for the fluid equations and the magnetic induction equation adopted from [20]. The fluid part of the vector, $\mathbf{U} = (\rho, \rho v_r, \rho v_\theta, \rho v_\phi r \sin\theta, e)^T$, reads as follows:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \mathbf{F} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\sin\theta \mathbf{G} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\mathbf{H} = \mathbf{S} \tag{1}$$

$$\mathbf{F} = \begin{pmatrix} \rho v_r \\ \rho v_r^2 + p + \frac{-B_{1r}^2 + B_{1\theta}^2 + B_{1\phi}^2}{2\mu} + \frac{-B_{1r}B_{0r} + B_{1\theta}B_{0\theta} + B_{1\phi}B_{0\phi}}{\mu} \\ \rho v_r v_\theta - \frac{B_{1r}B_{1\theta} + B_{1r}B_{0\theta} + B_{0r}B_{1\theta}}{\mu} \\ \left(\rho v_r v_\phi - \frac{B_{1r}B_{1\phi} + B_{1r}B_{0\phi} + B_{0r}B_{1\phi}}{\mu}\right) r\sin\theta \\ \left(\frac{1}{2}\rho v^2 + \frac{\gamma p}{\gamma-1}\right) v_r + \frac{B_{1\theta}}{\mu}(v_r B_\theta - v_\theta B_r) + \frac{B_{1\phi}}{\mu}(v_r B_\phi - v_\phi B_r) \end{pmatrix}$$

$$\mathbf{G} = \begin{pmatrix} \rho v_\theta \\ \rho v_r v_\theta - \frac{B_{1r}B_{1\theta} + B_{1r}B_{0\theta} + B_{0r}B_{1\theta}}{\mu} \\ \rho v_\theta^2 + p + \frac{B_{1r}^2 - B_{1\theta}^2 + B_{1\phi}^2}{2\mu} + \frac{B_{1r}B_{0r} - B_{1\theta}B_{0\theta} + B_{1\phi}B_{0\phi}}{\mu} \\ \left(\rho v_\theta v_\phi - \frac{B_{1\theta}B_{1\phi} + B_{1\theta}B_{0\phi} + B_{0\theta}B_{1\phi}}{\mu}\right) r\sin\theta \\ \left(\frac{1}{2}\rho v^2 + \frac{\gamma p}{\gamma-1}\right) v_\theta + \frac{B_{1r}}{\mu}(v_\theta B_r - v_r B_\theta) + \frac{B_{1\phi}}{\mu}(v_\theta B_\phi - v_\phi B_\theta) \end{pmatrix}$$

$$\mathbf{H} = \begin{pmatrix} \rho v_\phi \\ \rho v_r v_\phi - \frac{B_{1r}B_{1\phi} + B_{1r}B_{0\phi} + B_{0r}B_{1\phi}}{\mu} \\ \rho v_\theta v_\phi - \frac{B_{1\theta}B_{1\phi} + B_{1\theta}B_{0\phi} + B_{0\theta}B_{1\phi}}{\mu} \\ \left(\rho v_\phi^2 + p + \frac{B_{1r}^2 + B_{1\theta}^2 - B_{1\phi}^2}{2\mu} + \frac{B_{1r}B_{0r} + B_{1\theta}B_{0\theta} - B_{1\phi}B_{0\phi}}{\mu}\right) r\sin\theta \\ \left(\frac{1}{2}\rho v^2 + \frac{\gamma p}{\gamma-1}\right) v_\phi + \frac{B_{1r}}{\mu}(v_\phi B_r - v_r B_\phi) + \frac{B_{1\theta}}{\mu}(v_\phi B_\theta - v_\theta B_\phi) \end{pmatrix}$$

where $e = \frac{1}{2}\rho \mathbf{v}^2 + \frac{p}{\gamma-1} + \frac{1}{2}\mathbf{B}_1^2$ corresponds to the modified total energy density consisting of the kinetic, thermal, and magnetic energy densities (written in terms of $\mathbf{B_1}$). ρ is the mass density, $\mathbf{v} = (v_r, v_\theta, v_\phi)$ are the flow velocities in the frame rotating with the Sun, p is the thermal pressure, and $\mathbf{B} = \mathbf{B_0} + \mathbf{B_1}$ denotes the total magnetic field consisting of the time-independent potential magnetic field $\mathbf{B_0}$ and its time-dependent derived part, $\mathbf{B_1}$. Since $\mathbf{B_0}$ is constant with time, many terms near $\mathbf{B_0}$ on the right-hand side vanish. t and $\mathbf{r}$ are time and position vectors originating at the center of the Sun. $\mu = 4 \times 10^{-7}\pi$ is the magnetic permeability of free space, $\mathbf{g} = -\frac{GM_s}{r^3}\mathbf{r}$ is the solar gravitational force, $G = 6.673 \times 10^{-11}$ m^3 s^{-2} kg^{-1} is the gravitational constant, $M_s = 1.99 \times 10^{30}$ kg is the solar mass, and $|\Omega| = 2\pi/26.3$ rad day^{-1} is the solar angular speed. In our code, we allow the ratio of specific heats, γ, to vary from 1.05 to 1.5 along the heliocentric distance, r, according to [17], that is, $\gamma = 1.05$ for $r/R_s \leqslant 5, \gamma = 1.05 + 0.03(r/R_s - 5)$ for $5 < r/R_s \leqslant 20$, and $\gamma = 1.5$ for $r/R_s > 20$.

The source terms, $\mathbf{S} = (S_1, S_2, S_3, S_4, S_5)^T$, are generated from the polar geometrical factors, the Coriolis, centrifugal, and gravity forces, and volumetric heating source terms. Explicitly,

$$\mathbf{S} = \begin{pmatrix} 0 \\ \rho\frac{v_\theta^2 + v_\phi^2}{r} + \frac{2p}{r} + \frac{B_{1r}^2 + 2B_{1r}B_{0r}}{r\mu} - \rho\frac{GM_s}{r^2} + \rho\Omega\sin\theta(2v_\phi + \Omega r\sin\theta) + S_M \\ S_{3,1} + \rho\Omega\cos(2v_\phi + \Omega r\sin\theta) \\ -2\rho\Omega(v_\theta\cos\theta + v_r\sin\theta) r\sin\theta \\ \rho v_r\left(-\frac{GM_s}{r^2} + \Omega^2 r\sin^2\theta\right) + \rho v_\theta \Omega^2 r\sin\theta\cos\theta + S_E + v_r S_M \end{pmatrix}$$

where

$$S_{3,1} = \left(p + \rho v_\phi^2 + \frac{B_{1r}^2 + 2B_{1r}B_{0r} + B_{1\theta}^2 + 2B_{1\theta}B_{0\theta} - B_{1\phi}^2 - 2B_{1\phi}B_{0\phi}}{2\mu}\right)\frac{\cot\theta}{r} + \frac{1}{r}\left(\frac{B_{1\theta}B_{1r} + B_{1\theta}B_{0r} + B_{0\theta}B_{1r}}{\mu} - \rho v_r v_\theta\right)$$

Here, S_M and S_E stand for the momentum and energy source terms, which are responsible for the coronal heating and solar wind acceleration. Following [17], the source terms S_M and S_E are given as follows: