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Multiphase flow simulation with gravity effect in anisotropic porous media using multipoint flux approximation

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ABSTRACT

Numerical investigations of two-phase flows in anisotropic porous media have been conducted. In the flow model, the permeability has been considered as a full tensor and is implemented in the numerical scheme using the multipoint flux approximation within the framework of finite difference method. In addition, the experimenting pressure field approach is used to obtain the solution of the pressure field, which makes the matrix of coefficient of the global system easily constructed. A number of numerical experiments on the flow of two-phase system in two-dimensional porous medium domain are presented. In this work, the gravity is included in the model to capture the possible buoyancy-driven effects due to density differences between the two phases. Different anisotropy scenarios have been considered. From the numerical results, interesting patterns of the flow, pressure, and saturation fields emerge, which are significantly influenced by the anisotropy of the absolute permeability field. It is found that the two-phase system moves along the principal direction of anisotropy. Furthermore, the effects of anisotropy orientation on the flow rates and the cross flow index are also discussed in the paper.

1. Introduction

Multiphase flows in porous media are ubiquitous at all scales from large scale applications as in oil and gas subsurface reservoirs to small scale applications as in porous membranes, metallic foams, filters and many others. No wonder, therefore, the great deal of motivations among researchers to adapt a framework that could handle the complexity of the flow of multiphase systems in porous media. Bearing in mind that researchers are still unable to fully comprehend to a great satisfaction the flow of multiphase systems at pore scale, it is, therefore, expected that formulating the problem of multiphase flows in porous media is even harder. One notices that even if it might be possible to resolve all the details down to pore scale, it will still be challenging to consider large-scale domains, which is a consequence of the enormous computing power that would be required. This necessitates that a coarser framework be developed based on the continuum hypothesis. The mathematical techniques that are used to rigorously develop such framework have to start from the equations that are applicable at pore scale and scale it up to porous media continuum scale. Unfortunately, since the governing equations at the pore scale for multiphase system are complex enough and there are still unresolved issues (e.g., the moving contact line problem), it is extremely difficult to rigorously derive upscaled equations starting with equations defined at pore scale. Moreover, even if this is possible they are going to be unclosed and a set of tedious exercises will need to be carried out to suggest closures [1]. Therefore, there is a great deal of motivation among researchers to extend Darcy's law such that an expression for phase velocity may be obtained. Researches, in this field, suggest that all the complexities of multiphase flow in porous media are lumped into a single multiplicative scalar parameter; namely relative permeability.

In the context of subsurface rock formations, another level of complexity arise as a result of anisotropy of media properties, which is due to the geological processes that took place over the longer geologic time scale. As a consequence, the fluid flow direction will not be only dependent on the pressure gradient but also on the principal directions of anisotropy. All the geological features mentioned above deliver the challenges on the development of robust reservoir simulators. The standard two-point flux approximation (TPFA) finite difference method is unfortunately not capable of handling full-tensor permeability, which limits the use of this method in many of porous media applications. This has led to the emergence of what is called the multipoint flux approximation (MPFA). Two approaches have been introduced to arrive at the finite difference stencil associated with the MPFA.

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The first was proposed by Avatsmark et al. [2] and Aavatsmark [3] who introduced MPFA O-method based on the finite volume method for which the pressure at the mid edges (face centers in 3-D) are used to ensure the continuity of pressure and flux. There are several other types of MPFA such as MPFA L-method [4–6], MPFA U-method [7] and MPFA Z-method [8]. In addition, Edwards and Rogers [9] and Lee et al. [10] introduced the flux-continuous approach based on the framework of finite difference, which is equivalent to the work of Aavatsmark et al. [2] and Aavatsmark [3]. The second approach of MPFA is based on the mixed finite element point of view. Wheeler and Yotov [11] proved that MPFA could be derived from the lowest-order Brezzi–Douglas–Marini (BDM) mixed finite element through special quadrature rule. A trapezoidal quadrature rule is employed such that it allows the local velocity elimination and leads to cell-centered stencil for the pressure (cell-centered finite difference). Employing quadrature rules in evaluating integrals alleviates the needs to care about the specific form of approximating shape functions and instead the values of the functions at nodal points are used. The resulting algebraic system is symmetric and positive definite. This approach is so-called multipoint flux mixed finite element (MPFMFE). In the last decade, the implementation of MPFA into single-phase or multiphase flow models for 2-D and 3-D problems has been deeply discussed [12–16].

As indicated in its name, MPFA requires more point stencil than TPFA. For example, the divergence operator requires 9-point and 27-point stencil in 2-D and 3-D problems, respectively. This makes the construction of the matrix of coefficient a difficult task and prone to errors in terms of coding. Therefore, we apply a newly developed technique, the so-called experimenting pressure field approach. This technique generates the matrix of coefficient automatically within the solver routine. This is, tremendously, beneficial particularly when there are long expressions of discretized algebraic equations. This technique has been implemented successfully in many engineering applications involving anisotropy of media properties [14–19].

The purpose of this paper is to simulate the flow of multiphase system in anisotropic porous media by using the MPFA method combined with the experimenting pressure field approach. In addition, we are also interested in investigating the effect of gravitational force in driving fluid migration in anisotropic porous media. In this work, numerical experiments for different anisotropy scenarios consider the inflow of a fluid phase that has lower density than the existing fluid inside the domain. The density difference between the two phases induces buoyancy-driven upward flow of the lower density fluid.

This paper consists of five main sections: Section 1 discusses the background and motivation of the study including the literature review on the development of MPFA and the scope of this work. Section 2 presents the governing equations of the two-phase flow model as well as the effect of gravitational force on the horizontal flux. Section 3 describes the fundamental concept of the MPFA method followed by the numerical discretization. Section 4 demonstrates the numerical results of the considered scenarios. The relation between the anisotropy orientation and the cross flow index is also discussed. Finally, we conclude the study in the last section.

2. Modeling equations and workflow of simulation

The governing equations that describe two-phase flow in porous media include the mass conservation equation and Darcy's law. The mass conservation equation describes the balance between the inflow and outflow of mass through a specified domain. The balance principle of physical quantities in the form of partial differential equations is a consequence of adapting the continuum hypothesis in porous media [20–22]. Meanwhile, Darcy's law describes the relationship among the total volumetric flow rate of each phase with the potential (pressure) gradient [23]. The mass conservation equation is described by

$$\frac{\partial(\phi\rho_\alpha S_\alpha)}{\partial t} + \nabla\cdot(\rho_\alpha \mathbf{u}_\alpha) = q_\alpha, \quad \alpha = w, n \tag{1}$$

Here, $\phi, \rho_\alpha, S_\alpha, q_\alpha$, and $\mathbf{u}$ are the porosity, the fluid density (kg/m^3), the saturation, the sources/sinks (kg/m^{3}s), and the flux (m/s) of α-phase, respectively. The index of w refers to the wetting phase and n for the non-wetting phase. Darcy's law for the flux of each phase is given by

$$\mathbf{u}_\alpha = -\frac{k_{r\alpha}}{\mu_\alpha}K(\nabla p_\alpha - \rho_\alpha \mathbf{g}), \quad \alpha = w, n \tag{2}$$

where μ_α and p_α are the fluid viscosity (Pa · s) and the pressure (Pa) of α-phase, respectively, $\mathbf{g}$ is the gravitational acceleration (m/s^2), $\mathbf{K}$ is the absolute permeability tensor (m^2), and $k_{r\alpha}$ is the relative permeability. The permeability tensor, $\mathbf{K}$, is given by

$$\mathbf{K} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \tag{3}$$

The full-tensor permeability is assumed to be symmetric ($K_{xy} = K_{yx}$) and positive definite ($K_{xx}K_{yy} > K_{xy}^2$). The relative permeability, which is a function of phase saturation, describes how a fluid phase interferes the flow behavior of another fluid phase and vice versa [24]. There are several models for the relative permeability such as Brooks–Corey [25] and van Genuchten [26]. In this work, we use the relative permeability model of Brooks–Corey, which may be written in the form:

$$\begin{aligned} k_{rw}(S_\alpha) &= (1 - S_\alpha)^{\frac{2+3\lambda}{\lambda}} \\ k_{rn}(S_\alpha) &= S_\alpha^2\left[1 - (1 - S_\alpha)^{\frac{2+\lambda}{\lambda}}\right] \end{aligned} \tag{4}$$

where λ is the pore size distribution index and its values vary depending on the heterogeneity of the reservoir rock. The greater the value of λ, the more homogeneous the reservoir rock will be. The typical values for λ range between 0.2 and 3. The Brooks–Corey's model has been used in many two-phase flow studies [27–30].

The governing equations as given above are coupled and there is no explicit equation for the pressure. In this work, the Implicit Pressure-Explicit Saturation (IMPES) scheme is adapted in which an equation for the pressure is obtained as follows: let's define the total flux as the summation of the wetting phase flux and the non-wetting phase flux,

$$\mathbf{u}_t = \mathbf{u}_w + \mathbf{u}_n \tag{5}$$

Rewriting the total flux in (5) based on the Darcy velocity for each phase in (2) and neglecting capillarity, one obtains

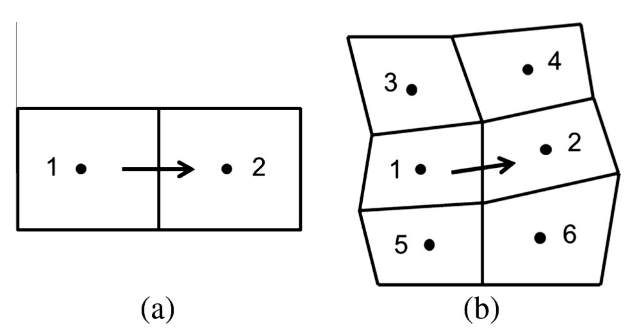

Fig. 1. Two-point flux (a) and the multipoint flux (b) approximations.