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An alternative second order scheme for curved boundary condition in lattice Boltzmann method

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ABSTRACT

An alternative scheme to implement the velocity Dirichlet boundary condition for curved boundary in the lattice Boltzmann (LB) method is developed. For inclined arbitrarily flat wall, the local second order boundary method (LSOBM) is proposed initially by Ginzbourg and D'Humières, and we further develop it to curved boundary, therefore a generalized LSOBM is achieved. In our boundary scheme, the unknown distribution functions at the boundary nodes are locally derived from the known ones by accessing the macroscopic physical information prescribed by the Dirichlet boundary conditions. Essentially, the unknown distribution functions are represented by a linear combination of the known ones, the corresponding coefficients depend on the macroscopic constraints on the boundary wall, the geometric information of the boundary nodes and the relaxation parameters. Unlike the previous curved boundary schemes, in which the boundary nodes are characterized by the intersected lattice links, a local curvilinear coordinate system associating with the curved boundary is adopted in the present scheme, and the boundary nodes are identified directly by their coordinates. Moreover, the present boundary scheme is second order accurate, as demonstrated in the theoretical derivations and also validated by two benchmark tests, the Taylor–Couette flow in-between rotating cylinders and the flow past an impulsively started cylinder.

1. Introduction

The lattice Boltzmann (LB) method, a promising alternative numerical technique to the traditional computational fluid dynamics (CFD), has been greatly developed over the last two decades, and a great success is achieved in simulating various complex fluids such as the multiphase flows [1,2], particulate flow [3], and the microfluidics [4,5]. The implementations of the boundary conditions have direct effect on the accuracy and numerical stability of the LB simulations. Therefore, the boundary scheme is crucial and becomes an attractive topic in developing LB method [6]. In the traditional CFD, the boundary conditions are defined with the macroscopic variables, whereas in the LB method, the primitive variables are distribution functions at the mesoscopic level, and no physically based boundary constraints for the distribution functions are provided. Thus, elaborate boundary schemes ought to be developed to determine the unknown distribution functions at the boundary nodes in accordance with the macroscopic physical boundary conditions [7–10].

The bounce back rule, originating from the lattice gas automata, has advantages in preserving naturally the mass conservation and implementing easily the programme. Therefore, the bounce back rule is widely used in simulating flows with especially complicated geometrics [7,10–12]. However, the standard bounce back rule is first order accurate, and it exhibits second order accuracy only when the actual non-slip wall is located off the grid point where the bounce back collision takes place [7,13,14]. Besides, for a moving boundary, an additional term representing the momentum transfer owing to the fluid–wall interaction ought to be introduced [6,7,13,14]. Unlike the bounce back rule, in which the boundary nodes locate outside the fluid domain to achieve second order accuracy, the boundary nodes are on the actual wall position in

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Nomenclature

f_α	distribution function
$f_\alpha^{(0)}$	equilibrium distribution function
$f_\alpha^{(neq)}$	non-equilibrium distribution function
τ	relaxation time
ξ_α	discrete velocity vector
w_α	weight coefficient
dx	spatial step
dt	time step
c	lattice speed
c_s	sound speed
$\mathbf{u}$	velocity vector
ρ	density
p	pressure
$\mathbf{\Pi}$	momentum flux tensor
$\mathbf{P}$	stress tensor
$\boldsymbol{\tau}$	deviatoric stress
ε	CE expansion parameter
ν	kinematic viscosity
Ma	Mach number
I	subsets of indices α
N	number of the elements in I
$\mathbf{g}_i$	covariant basis
u^i	contravariant velocity components
u_i	covariant velocity components
$u^i_{;j}$	covariant derivatives of the velocity
Γ^k_{ij}	Christoffel symbols of the second kind
$\mathbf{u}'$	given velocity on the boundary wall
r, θ	coordinates of the boundary node
r_0	radius of the bound wall
δ	normal distance $r - r_0$
r_1	radius of the inner cylinder
r_2	radius of the outer cylinder
β	geometric parameters of the Taylor–Couette flow
$\mathbf{e}_r, \mathbf{e}_\theta$	unit basis in the curvilinear coordinate system
Re	Reynolds number
ϵ_N	the L_2 norm error of the velocity fields
U	inlet velocity
D	diameter of the cylinder
T	dimensionless time
u_x, u_y	velocity components in the x, y directions
u_r, u_θ	velocity components in the normal, tangent directions
C_D	total drag coefficient
L/D	wakelength
C_{Dp}	pressure drag component

Superscript

(0)	equilibrium
(1)	the first order term of the CE expansion
loc	known distribution function components
in	unknown distribution function components
sol	normal projection of the boundary node on the boundary wall
use	the selected known distribution function components for computing the unknown macroscopic variables

Subscript

α	direction of the discrete velocity

another category of boundary schemes. In Skordos's scheme [15] and the velocity boundary condition proposed by Lätt and Chopard [9], the unknown distribution functions are computed from the first order Chapman–Enskog (CE) expansion, and the velocity gradients are approximated by a finite-difference scheme. In the moment-based boundary scheme proposed by Reis and Dellar [4], the unknown distribution functions are derived from a closure relation constructed by the prescribed moments pertaining to the hydrodynamic quantities, including density, momentum, and momentum flux, on the boundary wall. Another typical boundary scheme in this category is the extrapolation scheme [11,12], which originates from the traditional finite difference schemes and is applicable for various physical boundary conditions.

In dealing with curved boundary, there are two strategies in the LB method. One is the nonuniform grid using second order interpolations [16,17], and the geometric integrity is exactly preserved [8]. Another strategy is to maintain the regular Cartesian grid, and special boundary schemes ought to be developed for representing the curved boundaries. The bounce back rule is extended to the curved boundary by interpolating the distribution functions, and the bounce back collision around the intersection of the lattice link and the boundary wall is reproduced [8,13,18]. The boundary scheme proposed by Lätt and Chopard [9] is also applied to the curved boundary by approximating the macroscopic velocity at the boundary nodes with interpolation along the lattice links and computing the velocity gradients with finite difference stencils along the grid lines [18]. Besides, the extrapolation scheme is also extended to the curved boundaries by Guo et al. [19]. The unknown distribution functions at the boundary nodes are firstly split into their equilibrium and non-equilibrium parts. The macroscopic variables and the non-equilibrium part of the distribution functions are approximated by extrapolation, then, the equilibrium part are obtained from the approximated macroscopic variables. Moreover, the boundary-fitting scheme, which is proposed by Filippova–Hänel [20,21] and further improved by Mei et al. [8,22] (hereinafter referred to as FH scheme), provides another alternative for curved boundary treatments. In the FH scheme, the linear interpolation, involving of the post-collision distribution functions at the fluid node near the boundary wall and the fictitious equilibrium distribution functions at the neighboring solid node, is adopted to determine the unknown distribution functions at the boundary nodes. The interpolation coefficient depends on the extrapolation scheme, which determinate the velocity at the neighboring solid node. The FH scheme has been applied successfully in some practical flow problems, such as the gas particle flow in filters [20] and oscillations of laminae in viscous fluids [23].

The above-mentioned curved boundary schemes are mostly of second order accuracy. In 1996, a so-called local second order boundary method (LSOBM) for the LB FCHC model was proposed by Ginzbourg and D'Humières [7]. The LSOBM was applied to an arbitrarily inclined flat wall, and the error term is of third order. Especially, the distribution functions at the boundary nodes are derived by the second order CE expansion and the macroscopic velocity and its first and second order derivatives at the boundary nodes are obtained from a second order Taylor expansion. Instead of the finite difference approximation as in Skordos's scheme [15], the local known distribution functions are used to compute the unknown macroscopic variables necessary for the expression of the distribution function, and therefore, the unknown distribution functions at the boundary nodes are ultimately expressed as a linear combination of the known ones at the same nodes. As claimed by Mei et al. [8], the LSOBM is viewed as the most profound and rigorous theoretical treatment of the boundary conditions, however, insufficient attention is devoted to this scheme owing to its