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Effect of buoyancy ratio on unsteady thermosolutal combined convection in a lid driven trapezoidal enclosure in the presence of magnetic field

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ABSTRACT

In this paper we concentrate to investigate the double diffusive unsteady mixed convection flow in a trapezoidal enclosure in presence of magnetic field. The bottom wall of the enclosure is heated and concentrated uniformly (case-I) and non-uniformly (case-II) while the top wall is cooled and moved uniformly with a constant velocity. Both side walls are adiabatic and impermeable. The coupled governing equations for this phenomenon is solved numerically using weighted residual based Galerkin technique of finite element method (FEM) for Richardson's number ($\mathrm{Ri} = 0.1$–100) and Buoyancy ratio ($\mathrm{Br} = -10$ to 10) at time $\tau = 1.0$. Reynolds number, Prandtl number and Lewis number are fixed at 100, 0.71 and 10 respectively. Streamlines, isotherm lines and iso-concentration lines are used to show the result graphically for velocity, temperature and mass distribution respectively. Nusselt and Sherwood number values are presented graphically to show the heat and mass transfer rate from the bottom surface of the cavity.

1. Introduction

The buoyancy driven flow of temperature and concentration is known as double-diffusive convection. In the practical application, double-diffusive natural or mixed convection is evident frequently. Solar collectors, nuclear reactor, building heating and cooling, petroleum extraction, chemical catalytic reactor, geothermal reservoir, etc. are the practical application of double-diffusive convection. Many authors [1–7] considered double diffusive convection in different type of enclosure. Papanicolaou and Belessiotis [8] considered an asymmetric trapezoidal enclosure to study double-diffusive natural convection numerically. This type of enclosure encountered in greenhouse type solar stills. A multi-cellular flow field is observed inside the cavity and number of cells depends on the Rayleigh number for a fixed geometry and fixed Lewis number. A computational analysis of double-diffusive convection in a rectangular enclosure studied by Lee and Hyun [9] and Hyun and Lee [10] for both adding and opposing horizontal temperature and concentration gradients. In steady state, distinct flow regimes are seen for large variation of buoyancy ratio. The most important inconsistencies between the results of the opposing gradient cases and the cooperating gradient cases are exposed. The effect of horizontal temperature gradient and vertical solutal gradient on double-diffusive convection in a cavity was depicted numerically by Lee and Hyun [11]. Moukalled and Darwish [12] employed finite volume method to investigate double-diffusive natural convection in porous rhombic cross section. Numerical results indicate that the flow field is less affected by heat transfer than by the mass transfer for $\mathrm{Le} = 10$ and $\mathrm{Br} = 10$. However convection effects augmented with the enhancement of Darcy number, Rayleigh number and enclosure gap. A two dimensional computational assessment of combined double-diffusive convection and radiation in a square enclosure with semitransparent fluid is carried out by Moufekkir et al. [13]. The couple governing equations of the physical model are solved by a hybrid scheme with multiple relaxation time lattice Boltzmann and finite difference method. Heatlines and masslines are inclined in the center of the cavity. In presence of cooperating flow case, the flow is more stabilized but flows are found to be a catalyser of the multi-cellular structures. Ali Mchirgui et al. [14]

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Nomenclature

B_0	strength of the magnetic field, (Tesla)
Br	buoyancy ratio $\{\beta_c(c_h-c_l)/\beta_T(T_h-T_l)\}$
c	mass concentration (kg m^{-3})
c_p	specific heat (J kg^{-1} k^{-1})
c_h	high mass concentration (kg m^{-3})
c_L	low mass concentration (kg m^{-3})
C	dimensionless mass concentration $(c-c_l)/(c_h-c_l)$
D	mass diffusivity (m^2 s^{-1})
g	gravitational acceleration (m s^{-2})
Ha	Hartmann number $(B_0 L\sqrt{\sigma/\mu})$
k	fluid conductivity (W m^{-1} k^{-1})
L	cavity height (m)
Le	Lewis number (α/D)
Nu	average Nusselt number
p	dimensional pressure (kg m^{-1} s^{-2})
P	non-dimensional pressure $((p+\rho g y)L^2/\rho U_0^2)$
Pr	Prandtl number (ν/α)
Re	Reynolds number $(U_0 L/\upsilon)$
Ri	Richardson number $(g\beta_T(T_h-T_l)L/U_0^2)$
Sh	average Sherwood number
T	temperature (K)
T_h	heat source temperature (K)
T_l	Heat sink temperature (K)
(u, v)	velocity components (m s^{-1})
(U, V)	dimensionless velocity component $(U = u/U_0,\ V = v/U_0)$
(x, y)	dimensional coordinates (m)
(X, Y)	dimensionless coordinates $(X = x/L, Y = y/L)$

Greek symbols

α	thermal diffusivity (m^2 s^{-1})
β_T	coefficient of thermal expansion (K^{-1})
β_c	coefficient of mass expansion (m^3 kg^{-1})
μ	dynamic viscosity (kg m^{-1} s^{-1})
ν	kinematic viscosity (m^2 s^{-1})
θ	non-dimensional temperature $\{(T-T_l)/(T_h-T_l)\}$
ρ	density (kg m^{-3})
ρ^*	dimensionless density
σ	electrical conductivity (S m^{-1})
Ψ	dimensional stream function
ψ	dimensionless stream function
Ω	vorticity
ξ	dimensionless vorticity

Subscripts

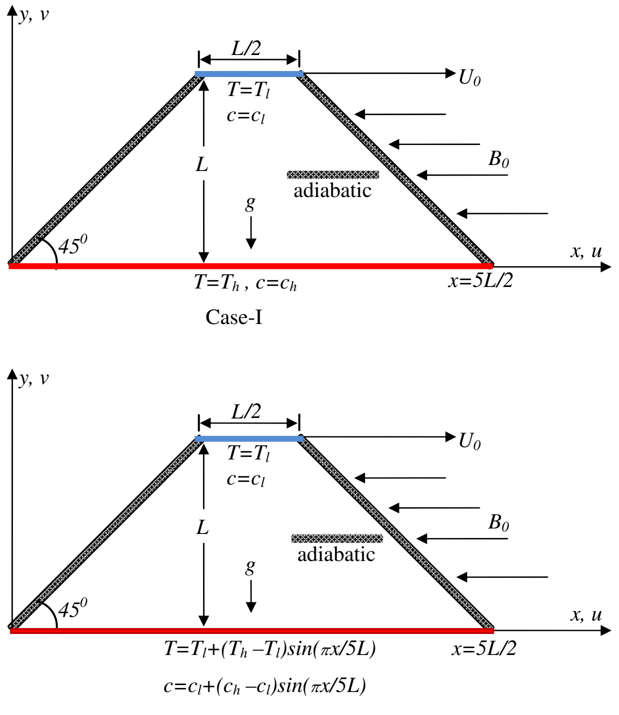
h	high (concentration/temperature)
l	low (concentration/temperature)

Abbreviation

MHD	magneto-hydrodynamic
CBC	convective boundary conditions

studied numerically the entropy generation in steady-unsteady double diffusive convection inside an inclined porous cavity. They showed that entropy generation reveals an oscillatory behavior for lower value of Darcy number. Minimum entropy generation is found at the aspect ratio 0.5 and 1, for Darcy number 10^{-2} and 10^{-4}, respectively.

Mixed convection in different type of enclosures [15–20] has received a sustained attention due to its wide range of application in many technological processes such as chemical process, lakes and reservoirs, air conditioning and thermal design of buildings and the cooling of electronic circuit boards etc. The effect of circumferential distribution of the wall heat flux on laminar mixed convection in a horizontal rectangular ducts studied by Chou [21]. Four different non-uniform heating conditions are applied. It is found that the flow and heat transfer characteristic influences significantly the strength and pattern of secondary flow. Also the asymptotic solutions are compared with the analytic solution and found good agreement. Cheng and Liu [22] considered differentially heated lid driven square cavity to investigate the effect of temperature gradient orientation on the fluid flow and heat transfer numerically using higher order compact scheme. The temperature gradient is imposed along the four side of the cavity. They found that both direction of temperature gradient and Richardson



Case-I

Case-II

Fig. 1. Schematic diagram for the problem with boundary conditions.

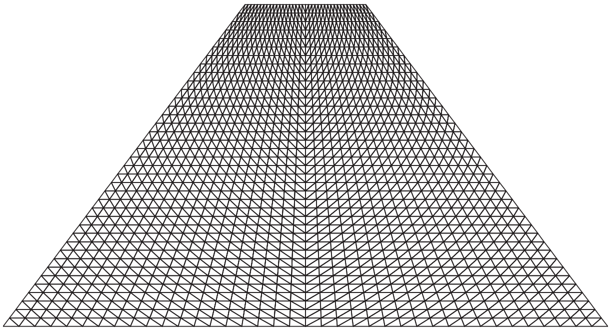


Fig. 2. Mesh of the trapezoidal enclosure considered in this work.