Computers & Fluids 107 (2015) 155–164

Contents lists available at ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

One-equation sub-grid scale model with variable eddy-viscosity coefficient

Javad Taghinia [a,*], Md Mizanur Rahman [a], Timo Siikonen [a], Ramesh K. Agarwal [b]

[a] Aalto University, School of Engineering, Espoo, Finland
[b] Washington University, Department of Engineering & Material Science, USA

ARTICLE INFO

Article history:
Received 29 October 2013
Received in revised form 10 October 2014
Accepted 24 October 2014
Available online 13 November 2014

Keywords:
SGS model
Eddy-viscosity coefficient
LES
Non-equilibrium
Single-filter
Hybrid time-scale

ABSTRACT

A one-equation sub-grid scale (SGS) model with a variable eddy-viscosity coefficient is developed for large-eddy simulation. The model coefficient is determined based on the shear and vorticity parameters accompanied by the hybrid time scale (T_t). The current model accounts for the SGS kinetic energy which is not considered in the dynamic Smagorinsky model (DSM). The eddy-viscosity coefficient preserves the anisotropic characteristics of turbulence in the sense that it is sensitized to non-equilibrium flows. In addition, it guarantees the positivity in the energy components. Unlike the original Smagorinsky model and DSM, the current SGS model does not need any *ad-hoc* damping function or clipping. The model is validated against well-documented flow cases, yielding predictions in good agreement with the direct numerical simulation (DNS) and experimental data. Comparisons indicate that the present model offers competitiveness with the DSM.

1. Introduction

In large-eddy simulation (LES), the flow field is decomposed into two components, namely, the prominent large scale and the sub-grid scale (SGS). The larger scale structures of the flow field are solved directly while the effects of the smaller scales which are smaller than the filter size are modeled. Typically, the grid itself is used as the low-pass filter, giving rise to the approach known as the implicit filter, which is the most commonly employed scheme in LES since no explicit filtering is applied to the governing equations. The SGS model plays a vital role in the performance of the LES method. The first SGS eddy-viscosity model is due to Smagorinsky [1]. This model has a few drawbacks that limit its application to some fluid flow problems. One of these shortcomings is that the model needs an empirical constant which varies for different flow problems. The second shortcoming is due not to reproducing correctly the decrease in turbulence level approaching the wall.

To overcome these limitations, Germano et al. [2] implemented a dynamic method in which the model coefficient is determined dynamically through the scale-similarity definition and the local-equilibrium hypothesis. The model coefficient thus obtained is a local value, varying in time and space over a fairly wide range with both negative and positive values. Although a negative coefficient and consequently negative eddy-viscosity is often interpreted as the flow of energy from the SGS eddies to the resolved eddies (referred to as back-scatter) and regarded as a desirable attribute of the dynamic Smagorinsky model (DSM), too large a negative eddy-viscosity causes numerical instability, eventually leading to an excessive level of numerical noise or even divergence of the numerical solution. To avoid this occurrence, the coefficient is simply clipped at zero. This method is slightly different from the usual practice in which the total viscosity (laminar viscosity + eddy-viscosity) is equated to zero, thus allowing a small SGS eddy-viscosity.

In principle, the main attention should be drawn to the calculation of so-called Smagorinsky constant (C_s). In the original SGS model, (C_s) is assigned a constant value which needs to be changed from one flow to another. In the dynamic version of the model, C_s (or C_d) is calculated based on variational methods, namely the least square minimization [3] or the Lagrangian method [4]. These methods produce a unique value for C_s from a system of five scalar equations, relating the anisotropic part of SGS turbulent stress tensor to the resolved strain-rate tensor. However, this procedure is proven to have a lack of accuracy, especially at a high Reynolds number for confined flows close to the wall region [5].

The value of C_s calculated by the dynamic approach is also incapable of reproducing the local sub-grid dissipative process. Nicoud and Ducros [6] proposed a new sub-grid scale model (i.e., WALE: wall-adapting local eddy-viscosity) for large eddy simulation in complex geometries with a constant coefficient in the eddy-viscosity

* Corresponding author.
E-mail address: javad.taghiniaseyedjalali@aalto.fi (J. Taghinia).

http://dx.doi.org/10.1016/j.compfluid.2014.10.014
0045-7930/

formulation. This model which is based on the square of the velocity gradient tensor accounts for the effects of both the strain-rate and the rotation rate of the smallest resolved turbulent fluctuations. It does not involve explicit filtering, averaging or clipping procedures, and the eddy-viscosity goes to zero in the vicinity of a wall so that neither a(dynamic) constant adjustment nor a damping function is needed to compute wall-bounded flows. The model produces zero eddy-viscosity in the case of a pure shear. Thus, it is possible to reproduce the laminar to turbulent process through the growth of linear unstable modes. Vreman et al. [7] developed a new SGS model with constant coefficients. In this model, the sub-grid dissipation vanishes for laminar flows or close to the walls, and it does not need any averaging or clipping for ensuring the numerical stability. Moreover, it requires no wall-damping function for the turbulent viscosity production which is difficult to formulate for complex flows with separation, curvature and rotation.

You and Moin [8] proposed a dynamic approach based on the work of Vreman et al. [7] to determine the model coefficient. They introduced a “global equilibrium” that assumes a global balance between the SGS dissipation and the viscous dissipation. In this model, the model coefficient is globally uniform and no *ad-hoc* clipping is needed for the numerical instability.

Zang et al. [9] formulated a mixed model in which the scale similarity model of Bardina [10] and dynamic Smagorinsky model of Germano [2] are combined. This model showed better predictions compared with the dynamic Smagorinsky model, however it experiences negative and highly fluctuating values for the model coefficients.

In this paper, a new model for calculating C_s (denoted C_k hereafter) is proposed in which the model coefficient is determined from the strain-rate and vorticity parameters. Therefore, it responds to both the shear and vorticity dominated flows that are far from equilibrium. The new coefficient also ensures realizability of the resolved normal stresses in question. Unlike the DSM, this model needs only a single filter making it more robust for use in majority of fluid flow problems. In addition, it requires no *ad-hoc* strategies for achieving the numerical stabilization. Finally, one can save some computational effort in the proposed model, since the test-filtering operation on the SGS stress is not needed.

2. Mathematical formulation

A spatial filter is employed in LES to separate the large scales from the small scales that are to be modeled [11]. A filtered variable in LES is expressed as:

$$f = \bar{f} + f_{sgs}, \qquad \bar{f} = \int_{R^3} \overline{G}(x; x')f(x')dx' \tag{1}$$

where $\overline{G}(x, x')$ is a low pass filter function. A filter operator can be of different types such as the top-hat filter, the Gaussian filter or the sharp Fourier cutoff filter. The top-hat and Gaussian filters give similar results; in particular, they both smooth the large-scale fluctuations as well as the small-scale ones, unlike the Fourier cutoff, which only affects the scales below the cutoff wave number [12]. Applying the spatial filter to incompressible Navier–Stokes equations and using the commutation characteristics, the LES equations yield:

$$\frac{\partial \bar{u}_j}{\partial x_i} = 0 \tag{2}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu \frac{\partial \bar{u}_i}{\partial x_j}\right) - \frac{\partial \tau_{ij}}{\partial x_j} \tag{3}$$

The *overbar* notation denotes the application of a top-hat filter and ν is the kinematic viscosity. On the right-hand side, an unresolved term τ_{ij} remains to be modeled. This term is analogous to the

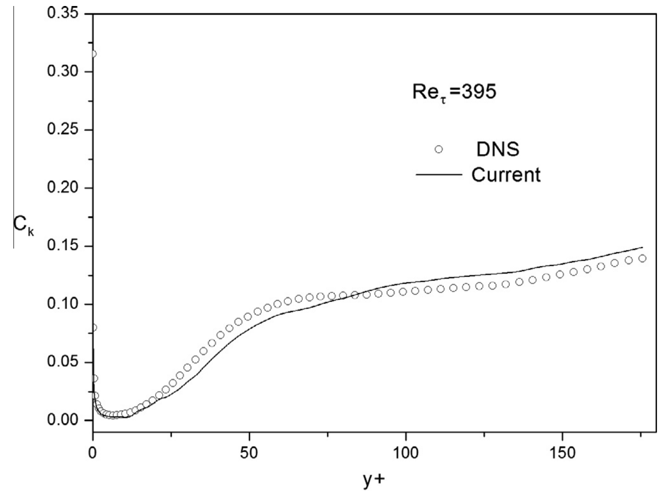

Fig. 1. C_k profile against DNS data.

Reynolds-stress tensor of RANS (Reynolds-averaged Navier–Stokes). Since in the LES formulation the larger length scales are resolved, it denotes the turbulent SGS stresses and hence, is smaller than its counterpart in RANS. The SGS stress tensor is defined as

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j \tag{4}$$

The role of the SGS model is to remove energy from the resolved scales. In LES, the small dissipative scales are not resolved accurately. Therefore, the SGS model is needed to account for the dissipation of turbulent kinetic energy to the viscous forces. Thus, the SGS models do not attempt at producing SGS stresses accurately but only accounts for their effect in a statistical sense. The unknown SGS turbulent stresses resulting from the filtering operation in Eq. (4) need a closure. Following the Boussinesq approximation, the relationship between the anisotropic part of the SGS stress tensor and the large-scale (i.e., resolved) strain-rate tensor can be expressed as:

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -2\nu_T \bar{S}_{ij}, \qquad \bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) \tag{5}$$

The isotropic part of stress tensor $(\frac{1}{3}\delta_{ij}\tau_{kk})$ is implicitly added to the pressure. The SGS eddy-viscosity ν_T is assumed to be a scalar quantity and is determined from the SGS kinetic energy k_{sgs} by the equation:

$$\nu_T = C_k \overline{\Delta}\sqrt{k_{sgs}} \tag{6}$$

where k_{sgs} is defined as

$$k_{sgs} = \frac{1}{2}\tau_{kk} = \frac{1}{2}(\overline{u_k u_k} - \bar{u}_k \bar{u}_k) \tag{7}$$

which can be obtained by contracting the subgrid-scale stress in Eq. (4). However, with the current model k_{sgs} is computed from its transport equation, given in Section 2.1. The grid-filter length (or width) $\overline{\Delta}$ is based on the cell volume:

$$\overline{\Delta} = (\Delta_1 \Delta_2 \Delta_3)^{\frac{1}{3}} \tag{8}$$

where Δ_1, Δ_2 and Δ_3 are the grid sizes in x, y and z directions, respectively. The eddy-viscosity coefficient C_k appearing in Eq. (6) is an indisputably flow-dependent quantity which can be readily computed as a scalar function of the invariants formed on the resolved strain-rate $\bar{S}_{ij}$ and vorticity $\overline{W}_{ij}$ tensors in question. The resolved strain-rate tensor $\bar{S}_{ij}$ is given in Eq. (5). The resolved $\overline{W}_{ij}$ is given by