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# A new flux splitting scheme for the Euler equations

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ABSTRACT

With the rapid development of the Computational Fluid Dynamics (CFD), schemes with higher levels of accuracy, robustness, and efficiency are in increasing demands. To achieve this goal, we propose a new scheme called E-AUSMPW in this paper. This scheme adopts the Zha–Bilgen splitting procedure by theoretical analysis and computes the convection flux like AUSMPW+. Moreover, it uses different methods to simulate the pressure flux's terms respectively. Series of numerical experiments show that E-AUSMPW is with a high level of robustness against shock anomalies. In addition, it is much more robust against the 'overheating phenomenon' than others. Besides these merits above, it is also with high accuracy and high efficiency orders in hypersonic heating predictions. Thus, E-AUSMPW is promising to be widely used to accurately and efficiently simulate both simple and complex flows.



## 1. Introduction

With the enormous advances in Computational Fluid Dynamics (CFD), numerical flux schemes have been widely regarded as matured. However, Liou argued that there were still fundamental problems needing much effort to resolve [1]. Nowadays, Central schemes [2,3], Flux Difference Splitting (FDS) schemes [4–6], Flux Vector Splitting (FVS) schemes [7,8], and AUSM-type schemes [9–13] are used widely. Among them, the Central schemes are known to be incapable of resolving intermediate characteristic fields accurately [1]. Moreover, they have strong dependencies on problem-dependent parameters. Thus, these Central schemes' applications are limited. FDS schemes are designed based on the Godunov's idea. Among them, Roe's FDS [4], HLLE [5], Osher's FDS [6] et al. win high praises because they can capture contact discontinuities accurately. Moreover, they are with high shock-capturing resolutions. However, they are inclined to show the 'carbuncle phenomenon' in certain problems [14]. To avoid this unphysical failing, people use an entropy fix [15,16] But a fix on the liner wave field is not a real entropy fix, because only the non-liner waves should be fixed to satisfy the entropy condition. In addition, it needs a detection procedure which usually involves a tuning coefficient. Improper coefficient's definition may broaden the shock wave profile and/or deteriorate the resolution [17]. The FVS schemes, such as Steger and Warming's FVS [7] and van Leer's FVS [8], can avoid these phenomena. Moreover, they are more robust and less time consuming than the FDS schemes. However, these schemes have accuracy problems in resolving shear layer region due to excessive numerical dissipation. These problems occur more seriously in supersonic and hypersonic flows. The AUSM-type schemes, proposed by Liou and other researchers, combine the merits of FDS and FVS. They split the Euler equations into a convective part and a pressure part. Through numerous researchers' efforts, these schemes are becoming more robust, accurate, and efficient. In addition, other FVS-type schemes with high accuracy and efficiency orders were developed by different Euler equations' splitting procedures [18–22].

In spite of the inspiring achievements made above, all these schemes are still faced with some unresolved problems, such as the shock anomaly and the 'overheating phenomenon' [1]. In this paper, we will give a theoretical analysis on the characteristic property of the AUSM-type schemes' splitting procedure. By comparing it with the other widely used splitting procedures theoretically, we adopt a distinct way to split the Euler equation. Also, by constructing the mass flux and the pressure flux respectively, we propose a new scheme called E-AUSMPW. Numerical results below will show that the E-AUSMPW scheme can be more robust against the shock anomaly and the 'overheating phenomenon'. In addition, it is with high accuracy and efficiency orders in hypersonic heating predictions.

This paper is organized as follows. In the second section, we will briefly overview the governing equations in the finite volume's form. The 3rd section will show theoretical analysis on the existing splitting procedures. Moreover, we will introduce the

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E-AUSMPWM's construction in the 4th section. To make a thorough analysis, we will compare it with Roe's FDS, AUSM+, and AUSMPW+ which are most widely used today by several numerical tests in Section 5. The last section contains concluding remarks.

## 2. Governing equations

NS or Euler equations are usually used to describe the motion of flows [23]. They can be written in the following form:

$$\frac{\partial \mathbf{q}}{\partial \mathbf{t}} + \frac{\partial \mathbf{f}_x}{\partial \mathbf{x}} + \frac{\partial \mathbf{f}_y}{\partial \mathbf{y}} + \frac{\partial \mathbf{f}_z}{\partial \mathbf{z}} = \mathbf{0} \tag{1}$$

Define $I_{ijk}$ as

$$I_{ijk} = [x_{i-1/2}, x_{i+1/2}] \times [y_{j-1/2}, y_{j+1/2}] \times [z_{k-1/2}, z_{k+1/2}]$$

After making integration over the cell $I_{ijk}$ we will get

$$\frac{d\bar{\mathbf{q}}_{i,j,k}}{dt} + \hat{\mathbf{f}}_{x,i+1/2} - \hat{\mathbf{f}}_{x,i-1/2} + \hat{\mathbf{f}}_{y,j+1/2} - \hat{\mathbf{f}}_{y,j-1/2} + \hat{\mathbf{f}}_{z,k+1/2} - \hat{\mathbf{f}}_{z,k-1/2} = \mathbf{0} \tag{2}$$

where

$$\bar{\mathbf{q}}_{i,j,k} = \frac{1}{\Delta x \Delta y \Delta z} \int_{z_{k-1/2}}^{z_{k+1/2}} \int_{y_{j-1/2}}^{y_{j+1/2}} \int_{x_{i-1/2}}^{x_{i+1.2}} \mathbf{q} dxdydz$$

$$\hat{\mathbf{f}}_{x,i+1/2} = \hat{\mathbf{f}}(\mathbf{q}^L_{i+1/2}, \mathbf{q}^R_{i+1/2})$$

$\bar{\mathbf{q}}_{i,j,k}$ represents the mean value of the cell while $\hat{\mathbf{f}}_{x,i+1/2}$ is the Riemann solver at the interface. Moreover, $\mathbf{q}^L_{i+1/2}$, $\mathbf{q}^R_{i+1/2}$ can be regarded as the values at the interface.

If high spatial accuracy is wanted, higher order interpolation should be combined with to compute $\mathbf{q}^L_{i+1/2}$, $\mathbf{q}^R_{i+1/2}$.

## 3. Characteristic analysis of the advection/pressure split procedures

Up to now, distinct splitting procedures have been proposed to split the Euler flux into a convective part and a pressure part. In this section, we will give a characteristic analysis of them and exhibit their physical characteristics.

### 3.1. The Liou–Steffen splitting procedure [9]

Liou and Steffen split the flux vector into a convective part and a pressure part as follows in 1D form:

$$\mathbf{f} = \mathbf{f}^L_c + \mathbf{f}^L_p \tag{3}$$

where

$$\mathbf{f}^L_c = \begin{pmatrix} \rho u \\ \rho uu \\ \rho uH \end{pmatrix}, \quad \mathbf{f}^L_p = \begin{pmatrix} 0 \\ p \\ 0 \end{pmatrix} \tag{4}$$

The Jacobian matrix of $\mathbf{f}^L_c$ is given as

$$\mathbf{A}^L_c = \begin{pmatrix} 0 & 1 & 0 \\ -u^2 & 2u & 0 \\ -\gamma uE + (\gamma-1)u^3 & \gamma E - \frac{3}{2}(\gamma-1)u^2 & \gamma u \end{pmatrix} \tag{5}$$

where its eigenvalues are as follows:

$$\lambda_1 = u, \quad \lambda_2 = u, \quad \lambda_3 = \gamma u \tag{6}$$

The Jacobian matrix of $\mathbf{f}^L_p$ is given as

$$\mathbf{A}^L_p = \begin{pmatrix} 0 & 0 & 0 \\ \frac{\gamma-1}{2}u^2 & -(\gamma-1)u & \gamma-1 \\ 0 & 0 & 0 \end{pmatrix} \tag{7}$$

where its eigenvalues are as follows:

$$\lambda_1 = -(\gamma-1)u, \quad \lambda_2 = 0, \quad \lambda_3 = 0 \tag{8}$$

From Eqs. (5)–(8), we can see that the convective part of the Liou–Steffen splitting procedure is in accord with the physical characteristic. However, its pressure part has one eigenvalue $(-(\gamma-1)u)$ which is not equal to 0. Also, it appears to show a tendency of backset, which is in contradictory with the real physical character. Thus, the robustness and the accuracy of the schemes based on such splitting procedure are influenced, despite the fact that the AUSM-type schemes have gained a considerable success.

### 3.2. The Zha–Bilgen splitting procedure [18]

Zha and Bilgen split the flux vector into a convective part and a pressure part as follows in 1D form:

$$\mathbf{f} = \mathbf{f}^Z_c + \mathbf{f}^Z_p \tag{9}$$

where

$$\mathbf{f}^Z_c = \begin{pmatrix} \rho u \\ \rho uu \\ \rho uE \end{pmatrix}, \quad \mathbf{f}^Z_p = \begin{pmatrix} 0 \\ p \\ pu \end{pmatrix} \tag{10}$$

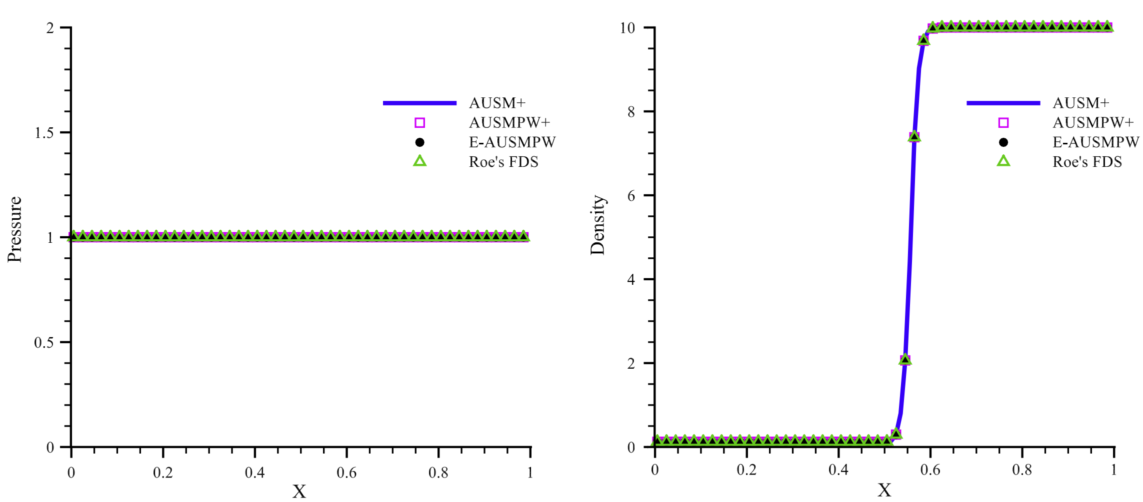


**Fig. 1.** Pressure and density distributions of a slowly moving contact discontinuity (mesh of 100 grid points).