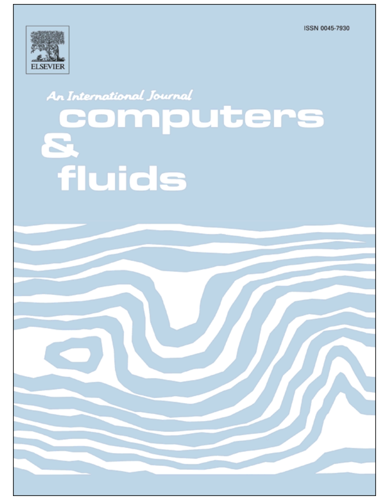

Multidomain, sparse, spectral-tau method for helically symmetric flow

M. Beroiz, T. Hagstrom, S.R. Lau, R.H. Price

PII: S0045-7930(14)00233-3
DOI: http://dx.doi.org/10.1016/j.compfluid.2014.05.028
Reference: CAF 2576

To appear in: *Computers & Fluids*

Received Date: 20 January 2014
Revised Date: 23 May 2014
Accepted Date: 24 May 2014

Please cite this article as: Beroiz, M., Hagstrom, T., Lau, S.R., Price, R.H., Multidomain, sparse, spectral-tau method for helically symmetric flow, *Computers & Fluids* (2014), doi: http://dx.doi.org/10.1016/j.compfluid.2014.05.028

MULTIDOMAIN, SPARSE, SPECTRAL-TAU METHOD FOR HELICALLY SYMMETRIC FLOW

M. BEROIZ, T. HAGSTROM, S. R. LAU, AND R. H. PRICE

Abstract. We consider the application of a multidomain, sparse, and modal spectral-tau method to the helically reduced Navier Stokes equations describing pipe flow. This work (i) formulates the corresponding modal approximations, (ii) describes improved boundary conditions for the helically reduced equations, and (iii) constructs iterative solutions of the corresponding elliptic problem that arises in the reduction. Regarding (iii), we also present and test a method for preconditioning the matching conditions between subdomains, a method based on statistical sampling and the interpolative decomposition. Although the following application is only discussed in our concluding section, a partial motivation for this work has been our ongoing development of similar spectral methods for the construction binary neutron star spacetimes.

1. Introduction and preliminaries

1.1. **Introduction.** Spectral methods are well-developed for the solution of complex flow problems in Cartesian, cylindrical, and spherical domains; see, for example, the monographs [1, 2]. However, important flow structures in pipes and jets often exhibit exact or approximate helical symmetry. Consider, for example, the travelling wave solutions constructed by Gertsenshtein and Nikitin [3], Pringle and Kerswell [4], and Smith and Bodonyi [5].

In this paper we develop an efficient spectral-tau method for solving the helically reduced Navier Stokes equations describing spiral-wave pipe flow (which we often informally refer to as the "pipe problem"). Applications will be considered in subsequent works. The techniques we apply in this paper to iteratively solve the helically reduced Navier-Stokes equations are as follows: (i) tau-integration sparsification [7, 6], (ii) block Jacobi preconditioning adapted to our modal approximations, and (iii) interpolative decomposition via stochastic sampling [8]. The following paragraphs briefly review each of these techniques. Following this review, we discuss our approach in the context of modern preconditioning techniques [9, 10] for elliptic equations.

Spectral tau methods [11] constitute a basic approach for approximation of differential equations. Differential operators are realized, as is standard for a modal spectral method, via recursion relations applied to the modal coefficients. In a tau method, the spectral basis functions are not adapted to the particular boundary conditions enforced in the problem; rather, supplementary relations between the modal coefficients (the "tau conditions") are introduced to enforce the boundary conditions. Tau integration-sparsification [7, 6] (called "integration preconditioning" in these references) is a particular tau formulation in which differential operators are undone using integration realized through recursion relations. For certain operators this formulation results in sparse matrices. It turns out that the integration step also reduces the number of equations, so that there are fewer equations than the

Corresponding author: S. R. Lau. Email: lau@unm.edu, Phone: +15052774613, Fax: +15052775505.