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# Computational and experimental studies of rapid free-surface granular flows around obstacles

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A B S T R A C T

The study of the rapid free-surface granular flows that are driven under gravitational acceleration has attracted much attention in recent years. This is not only because such flows occur in many industrial and natural examples, but also because their transport mechanisms can be observed through small-scale lab experiments, modeled with continuum theories, and simulated by computers. When granular particles rapidly propagate around obstacles, the resulting phenomena – shock waves, particle free regions or granular vacua, expansion fans and stagnation zones – are of particular theoretical and practical importance. In this paper we develop a computational method for a hydraulic-type avalanche model that is able to simulate such phenomena. It numerically solves the avalanche model over a structured grid by including the topography of the obstacles. Different finite-difference TVD methods based on the non-oscillatory central (NOC) scheme are tested in computation to compare the resolution of the shock waves when granular flows propagate against an oblique wedge. This is also involved in the choice of the limiters, but which are shown rather insensitive in such computations. A level set formulation is coupled into the governing equations to follow the evolution of the boundaries of the granular vacua. It is tested in a situation when granular particles flow around a circular cylinder, where bow shock waves, granular vacua, expansion fans and stationary zones are all captured in the computation. These results agree well and consistently with the laboratory experiments.



## 1. Introduction

Gravity-driven granular avalanches occur over a vast range of scales both in industry and in the natural environment. Snow avalanches, volcanic flows, rock-falls, pyroclastic flows, lahars and debris-flows are examples of natural hazards in geophysical scales, and smaller-scale examples are usually seen in industrial processes such as in the mining, bulk-chemical, pharmaceutical and food industries. In recent years the study of granular flows has attracted much attention because important phenomena such as shock waves, expansion fans, stationary zones and granular vacua are often observed particularly when a granular avalanche flows past an obstacle or over complex topography such as mountainous terrains. The study of these phenomena are of particular importance not just for gaining insight into associated theories and methodological approaches, but for practical uses as well. For example, knowing the avalanche thickness jump across a shock wave generated around a defensive infrastructure may determine a minimum requirement of the height of such infrastructure for stopping or diverting the avalanche from overtopping it; knowing the shape of a granular vacuum formed behind an obstacle could suggest the effectiveness of it as to divert the incoming avalanche.

The concept of granular shock wave, or *granular jump*, was initiated by Savage [23] when he observed a stationary jump upstream of a splitter plate. Gray and Hutter [7] studied upslope traveling normal shocks in pattern formation experiments on heaps and in rotating drums. Gray et al. [9] showed that the traveling normal shock was the granular equivalent of a *hydrodynamic bore*. Their experiments showed that the simple hydraulic analysis was accurate to within 10% of the observed shock heights and propagation speeds. Stationary oblique shocks were also observed in [9] when granular avalanches were deflected by wedge shaped obstacles. These studies suggest that the hydraulic type avalanche models may be effective to compute shallow granular flows. Such models were first proposed in Russia in the 1960s [11,14,5] to model snow avalanches, but their work did not catch on in the west till recently. Gray et al. [9] generalized the hydraulic model to investigate granular flows past obstacles that have sloped sides or walls normal to the chute, but the effect of the obstacles was treated as topographic gradients and incorporated into the source terms.

Nowadays, numerical computations of shock waves have built on much more sophisticated schemes. In the 1980s the concept of TVD (Total Variation Diminishing) [12] was introduced in shock

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capturing techniques to replace traditional high-resolution schemes. Such numerical approaches can be classified into two types: the upwind TVD schemes based on the Godunov method; and the central TVD schemes based on the Lax-Friedrichs method. The upwind TVD schemes have been widely used in the computational gasdynamics, where the approximate Riemann solvers are applied to follow the characteristics. The study of the central TVD schemes did not become popular until Nessyahu and Tadmor [19] developed a second-order Non-Oscillatory Central (NOC) TVD scheme in 1990 and this was followed by a number of improvements (see e.g. [1,13,15,16,28]). The central TVD schemes are regarded as a universal approach as they require no details of the characteristics and the Riemann solvers, and are applied in recent years to simulate rapid free-surface granular flows. Gray et al. [9] used a TVD Lax-Friedrichs scheme to simulate upslope traveling normal shocks. Tai et al. [27] used the NOC scheme to simulate the evolution of one-dimensional parabolic-cap similarity solutions [22]. Denlinger and Iverson [4] applied a Riemann solver algorithm to simulate variably-fluidized granular flows across three-dimensional terrains. Pitman et al. [21] used an adaptive mesh Godunov solver to simulate granular avalanches and landslides over a realistic terrain from digital elevation model data. In particular, Gray et al. [9] used the same NOC scheme as [27] to simulate the evolution of two-dimensional avalanches past various shaped obstacles, where both attached and detached shocks were captured. In dealing with the obstacles, they explicitly incorporated the topography of the obstacle into the source terms. Gray and Cui [10] applied classical oblique shock theory, small scale experiments and numerical simulations to investigate how weak, strong and detached shock waves were generated by a wedge. The same numerical method was further used by Cui et al. [3] to capture shocks for snow avalanches on realistic topography from a deflecting dam in Flateyri, Iceland, where good order of magnitude agreement was shown between the simulation and field observation for the avalanche track, the avalanche thickness across the shock and the run-out region.

As an avalanche propagates past an obstacle a particle-free region – a granular vacuum – is often generated at the lee-side. The vacuum boundary marks the position where the avalanche thickness $h$ first approaches zero and it is of considerable interest to accurately track the boundary from the computation. Tai et al. [27] used marker points to track the moving front in their one-dimensional simulations, but would find it infeasible if simulating two-dimensional flows. Here we intend to develop a level set formulation that fits naturally into the governing equations. While many contributions have been devoted into the level set methods (e.g. [20,25,2,17,18,24,26]), our approach is mainly based on the idea of Mulder et al. [18] who embedded a level set formulation to the system of conservation laws for compressible gas dynamics. They found that the coupling of the level set in the governing equations works better when there is a jump in the normal velocity across the interface. We shall experiment with a similar approach in our computation for the flow around a circular cylinder where a granular vacuum forms.

## 2. Governing equations

### 2.1. Granular avalanche model

To simulate granular avalanches around obstacles a dimensionless hydraulic-type model proposed in [5,9,11,14] is adopted. This model is set up in a fixed Cartesian coordinate system, namely, $Oxyz$, where the $x-$axis is along the downslope direction at an inclination angle $\zeta$ to the horizontal, the $y-$axis is along the lateral cross-slope direction, and the $z-$axis points upward according to the right-hand rule. The velocity $\boldsymbol{u}$ has components $(u, v, w)$ in each of these directions. The depth-integrated non-dimensional mass and momentum balances are thus given as

$$(h)_t + (hu)_x + (hv)_y = 0, \tag{1}$$

$$(hu)_t + (hu^2)_x + (huv)_y + \left(\frac{1}{2}h^2\cos\zeta\right)_x = hS^x, \tag{2}$$

$$(hv)_t + (huv)_x + (hv^2)_y + \left(\frac{1}{2}h^2\cos\zeta\right)_y = hS^y, \tag{3}$$

where $h$ is the thickness, $\zeta$ is the inclination angle of slope, and the subscripts "$x$" and "$y$" denote the derivatives, respectively. The source terms of the right-hand side are

$$S^x = \sin\zeta - \mu(u/|\boldsymbol{u}|)\cos\zeta, \tag{4}$$

$$S^y = -\mu(v/|\boldsymbol{u}|)\cos\zeta, \tag{5}$$

where $\sin\zeta$ represents the downslope component of the gravitational acceleration and $\mu$ is the coefficient of the basal Coulomb friction. Note, the above equations are based on a downslope basal topography of flat surface. Gray et al. [8] extended these equations over complex basal topography by including the variation of the basal surface in both downslope and lateral directions, which were successfully used by Cui et al. [3] for simulating snow avalanche development over a real geophysical terrain in Flateyri of Iceland.

The governing equations (1)–(3) are formulated in non-dimensional form where the variables have been non-dimensionalized by the scalings

$$\tilde{h} = Lh, (\tilde{x}, \tilde{y}) = L(x, y), (\tilde{u}, \tilde{v}) = \sqrt{Lg}(u, v), \quad \tilde{t} = \sqrt{(L/g)}t, \tag{6}$$

where the tildered variables are dimensional and $g$ is the constant of gravitational acceleration. The velocity scaling $\sqrt{Lg}$ is based on that of Savage and Hutter [22] who identified that the motion of the avalanche was primarily governed by free-fall of the grains rather than surface gravity waves, which would imply the scaling $\sqrt{Hg}$. Savage and Hutter's scaling identifies that the dominant balance in equation (2) is between the acceleration and the source terms. The depth-averaged pressure, which is multiplied by a factor $\varepsilon = H/L$, plays a lesser important role, while higher order terms are neglected (see e.g. [8,9]). Thus equations 1, 2 to 3 imply that $\varepsilon = 1$.

In our study, we let $L = 0.03$ m, the diameter of the circular cylinder used in the experiment, for all length scalings. It follows that the velocities are scaled by $\sqrt{gL} = 0.54\ \mathrm{ms}^{-1}$ and time is scaled by $\sqrt{L/g} = 0.055$ s. The Froude number

$$\mathrm{Fr} = |\bar{\boldsymbol{u}}|/c, \tag{7}$$

is defined as the ratio of the flow speed $|\bar{\boldsymbol{u}}|$ to the wavespeed $c = \sqrt{h\cos\zeta}$. The avalanche is described as being supercritical if $\mathrm{Fr} > 1$, critical if $\mathrm{Fr} = 1$ and subcritical if $\mathrm{Fr} < 1$. Note that the equations , 2 to 3 are closely linked to the equations of isentropic gas dynamics with equivalent specific heat ratio $\gamma = 2$, where the role of the Froude number replaces that of the Mach number. The major difference is the presence of the source terms (4) and (5). One of the important properties of the gas dynamics equations is that the momentum equations are trivially satisfied, and the velocity is arbitrary, when the gas density is identically zero. Despite the presence of the source terms, the avalanche equations have the same degeneracy, indicating that grain free regions, or granular vacua, arise naturally from the theory.

### 2.2. Level set representation

We intend to construct an equation for a function $\phi(x, t)$ which contains the embedded motion of the two-dimensional hypersurface $\Gamma(t)$ as the level set $\phi = 0$. Let $x(t)$ be the path of a point on the propagating front then