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Application of the meshless finite volume particle method to flow-induced motion of a rigid body

R.M. Nestor [1], N.J. Quinlan *

Mechanical Engineering, College of Engineering and Informatics, National University of Ireland Galway, Ireland

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ABSTRACT

We present a new approach to numerical modelling of incompressible flow of fluid about an elastically mounted rigid structure with large body motions. The solution is based on the Finite Volume Particle Method (FVPM), a meshless generalisation of the mesh-based finite volume method. The finite volume particles are allowed to overlap, without explicit connectivity, and can therefore move arbitrarily to follow the motion of a wall. Here, FVPM is employed with a pressure projection method for fully incompressible flow coupled with motion of a rigid body. The developed extension is validated for Vortex-Induced Vibration (VIV) of a circular cylinder in laminar crossflow. To minimise computational effort, non-uniform particle size and arbitrary Lagrangian–Eulerian particle motion schemes are employed, with radial basis functions used to define the particle motion near the cylinder. Close agreement is demonstrated between the FVPM results and a reference numerical solution. Results confirm the feasibility of FVPM as a new approach to the modelling of flow with strongly coupled rigid-body dynamics.

1. Introduction

Many important problems in fluid dynamics are dominated by moving boundaries. Examples include the heart and blood vessels, internal combustion engines, animal flight and vortex-induced vibration of slender elastic structures. In classical computational fluid dynamics based on boundary-conforming meshes, special treatments (such as mesh deformation, remeshing, overset meshes and immersed boundaries) are required for flows of this kind. In meshless methods, in contrast, the computational nodes or particles are free to move in response to boundary motions, since their connectivity need never be specified. Therefore, it appears that meshless methods can avoid the difficulties presented by a mesh with moving boundaries.

There is now a significant body of work on the meshless method smoothed particle hydrodynamics (SPH) demonstrating validated applications in a range of applications, notably in free-surface flow. SPH was first applied to free-surface flow by Monaghan [1] using a weakly compressible approach. Incompressible free-surface SPH methods were developed by Cummins and Rudman [2] using a pressure projection, and by Shao and Lo using a density-invariant formulation [3]. A comprehensive review is given by Monaghan [4].

The relatively new finite volume particle method (FVPM) is a meshless generalisation of the classical mesh-based finite volume method which, in principle, avoids some limitations of other meshless methods. The central idea of FVPM, and the main difference between it and the mesh-based finite volume method, is the definition of an interface area between overlapping finite volume cells (particles), in contrast with the contiguous but strictly non-overlapping finite volume cells in a mesh. Since finite volume particles may overlap arbitrarily, there is no need to determine or maintain connectivity information. They can move in any manner, as long as fluxes due to the particles' motion are accounted for. This makes it straightforward to accommodate moving boundaries. Where a particle is truncated by a boundary, a particle-boundary interface area is defined, enabling boundary flux to be computed. FVPM was introduced by Hietel et al. [5]. It was subsequently analysed by Junk and Struckmeier [6] and Junk [7], proving consistency of the scheme, and rigorously establishing FVPM as a generalisation of the finite volume method. Keck and Hietel [8] implemented a pressure projection scheme for fully incompressible flow. Improved methods for the particle interface area calculation were proposed by Hietel and Keck [9] and Teleaga [10]. Nestor et al. [11] extended FVPM to second-order spatial accuracy and viscous flows, and Teleaga [12,10] and Nestor and Quinlan [13] applied the method to moving-boundary problems.

FVPM embodies some valuable properties of the finite volume method without sacrificing the flexibility of a meshless method for moving boundaries and interfaces. Boundary conditions are implemented straightforwardly by prescription of fluxes from the

* Corresponding author.
E-mail address: nathan.quinlan@nuigalway.ie (N.J. Quinlan).
[1] Current address: MCS Kenny, Parkmore, Galway, Ireland.

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boundary to the particle. FVPM possesses proven theoretical consistency [6], and local conservation is exact, regardless of variation in particle size. In addition, a wide range of established finite volume techniques (e.g. Riemann solvers) can be directly incorporated in FVPM. However, there have been few applications of FVPM to date, and most have been restricted to simple benchmark problems.

This article is concerned with development and validation of FVPM for a more challenging problem in which full advantage can be taken of the meshless formulation. We describe the extension of the FVPM to flow-induced motion of a rigid structure (henceforth referred to as rigid-body FSI) for incompressible flow. This extension is validated for Vortex-Induced Vibration (VIV) of a circular cylinder in cross-flow, a problem involving coupling of fluid dynamics with a rigid body undergoing large displacements. The motivation for this study is to validate FVPM on a well-studied fluid–structure interaction problem, in advance of applications involving more complex rigid-body dynamics and/or elastic bodies.

The FVPM formulation is described in Section 2. The rigid-body FSI extension of FVPM is presented in Section 3 and a novel ALE particle motion scheme for FVPM is presented in Section 4. Results are presented in Section 5 from FVPM simulations for crossflow over a circular cylinder vibrating with prescribed motion and a freely vibrating cylinder. The FVPM results are compared with reference solutions from the literature throughout.

2. The finite volume particle method

2.1. FVPM formulation

The semi-discrete form of the FVPM for a conservation law is [5,12]

$$\frac{d}{dt}(V_i\mathbf{U}_i) = -\sum_{j=1}^{N}\boldsymbol{\beta}_{ij}\left(\mathcal{F}(\mathbf{U}_i,\mathbf{U}_j)\right) - \boldsymbol{\beta}_i^b\mathcal{F}_i^b, \tag{1}$$

where t is time, V_i is the volume of particle i, and $\mathbf{U}$ is the vector of conserved variables. The numerical flux $\mathcal{F}(\mathbf{U}_i,\mathbf{U}_j)$ is an approximation to $\mathbf{F}_{ij} - \overline{\mathbf{U}}_{ij}\bar{\dot{\mathbf{x}}}_{ij}$, where $\overline{\mathbf{U}}_{ij}$ and $\bar{\dot{\mathbf{x}}}_{ij}$ are averages of the conserved variables and particle transport velocity, respectively, of particle i and its neighbour j. The superscript b denotes boundary terms. The element of FVPM which differentiates it from the classical finite volume method is the particle interaction vector, defined by

$$\boldsymbol{\beta}_{ij} = \int_{\Omega}\frac{W_i\nabla W_j - W_j\nabla W_i}{\left(\sum_k W_k\right)^2}\,d\mathbf{x}, \tag{2}$$

where $W_i = W(\mathbf{x}-\mathbf{x}_i(t), h)$ is a compactly supported kernel function for particle i, centred at $\mathbf{x}_i$. The compact support radius is $2h$, where h is called the smoothing length, in keeping with the SPH convention. The quantity $\boldsymbol{\beta}_{ij}$ is precisely analogous to the cell face normal area vector which weights intercell fluxes in the classical finite volume method [7]. The particle interaction vectors are evaluated by numerical integration and corrected by the procedure of Teleaga [10] to ensure the condition $\sum_j\boldsymbol{\beta}_{ij} = 0$ (analogous to the condition that a cell surface is closed in traditional finite volume methods) is exactly satisfied.

Interparticle fluxes are computed using a MUSCL reconstruction from particle barycentres to particle–particle interfaces, as described by Nestor et al. [11]. The reconstruction is based on a consistency-corrected SPH estimate of gradients [14] at the particle barycentre. The HLL Riemann solver [15] is then used to approximate the interparticle inviscid momentum fluxes.

One valuable property of FVPM, exploited in the present work, is that particle size may be spatially non-uniform. That is, neighbours i and j can have different support radius $2h$. In the form used here, h_i is constant in time, although the case $h = h(t)$ may also be treated with an additional term involving dh/dt [5]. A second-order explicit Runge–Kutta scheme is used for temporal discretisation of Eq. (1).

For full details of FVPM, the reader is referred to Hietel et al. [5]. The implementation in the present work follows the details given by Nestor et al. [11,13], except where stated otherwise.

2.2. Fully incompressible solution methodology

FVPM has previously been applied to fully incompressible flow problems by several authors [8,13,16]. The pressure projection algorithm of Chorin [17], adapted for SPH by Cummins and Rudman [2], is used to achieve the fully incompressible flow solution. The algorithm can be summarised (using a first-order explicit temporal discretisation for brevity) by the following sequence of steps:

$$V_i^{n+1} = V_i^n + \Delta t\frac{dV_i^n}{dt}, \tag{3}$$

$$\mathbf{U}_i^* = \frac{1}{V_i^{n+1}}\left(V_i^n\mathbf{U}_i^n + \Delta t\frac{d(V_i\mathbf{U}_i)^{*,n}}{dt}\right), \tag{4}$$

$$\mathbf{x}_i^{n+1} = \mathbf{x}_i^n + \Delta t\dot{\mathbf{x}}_i^n, \tag{5}$$

$$\nabla^2 p_i^{n+1} = \frac{\rho}{\Delta t}\nabla\cdot\mathbf{u}_i^*, \tag{6}$$

$$\mathbf{u}_i^{n+1} = \mathbf{u}_i^* - \frac{\Delta t}{\rho}\nabla p_i^{n+1}, \tag{7}$$

where $d(V_i\mathbf{U}_i)^{*,n}/dt$ is computed from Eq. (1) without the pressure term in the flux function $\mathcal{F}$. The algorithm consists of a preliminary time advance of the momentum equation (disregarding the pressure terms), which yields the momentum $\mathbf{U}^*$ in Eq. (4) that is not guaranteed to satisfy the divergence-free velocity condition. The pressure solution at time $n + 1$ is computed in Eq. (6). The velocity is corrected in Eq. (7) so that the divergence-free velocity condition is satisfied.

Care must be taken when developing the discrete form of the pressure Poisson equation (Eq. (6)) to ensure that the discrete scheme does not admit spurious checkerboard solutions for the pressure. An appropriate choice for the discrete Laplacian and divergence operators is described by Nestor and Quinlan [13].

In the present work, the solution to the discretised pressure Poisson equation in Eq. (6) is obtained with the LASPACK implementation of the GMRES algorithm [18].

2.3. Boundary conditions

A significant advantage of FVPM over other mesh-free methods is that boundary fluxes can be prescribed straightforwardly wherever a particle is truncated by a boundary. The discretisation of these terms allows for a straightforward enforcement of boundary conditions in terms of a boundary flux and a geometric interaction vector. Following Keck [16] and Keck and Hietel [8], the boundary coefficient $\boldsymbol{\beta}_i^b$ in Eq. (1) may be computed from

$$\boldsymbol{\beta}_i^b = -\sum_{j=1}^{N}\boldsymbol{\beta}_{ij}. \tag{8}$$