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Turbulence structure and budgets in curved pipes

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ABSTRACT

Turbulent flow in curved pipes was investigated by Direct Numerical Simulation. Three curvatures δ (pipe radius a/curvature radius c) were examined: $\delta = 0$ (straight pipe), simulated for validation and comparison purposes; $\delta = 0.1$; and $\delta = 0.3$. The friction velocity Reynolds number (based on the pipe radius a) was 500 in all cases, yielding bulk Reynolds numbers of ~17,000, ~15,000 and ~12,000 for $\delta = 0$, 0.1 and 0.3, respectively. The computational domain was ten pipe radii in length and was resolved by up to 20×10^6 hexahedral finite volumes. The time step was chosen equal to a wall time unit; 1 Large Eddy TurnOver Time (LETOT) was thus resolved by 500 time steps and simulations were typically protracted for 20 LETOT's, the last 10 being used to build turbulence statistics and budgets.

In curved pipes, time mean results showed Dean circulation and a strong velocity stratification in the curvature direction, as in laminar flow; turbulent fluctuations were highest in the outer bend region, whereas the flow near the inner wall was almost laminar. Significant turbulence levels were confined to a near-wall layer narrower than in the straight pipe. Near-wall streamwise velocity and wall shear stress exhibited the streak structure typical of turbulent channel flows only along the outer wall, while on the inner wall they exhibited a flat and low-level distribution. As the curvature increased, fluctuations in the plane of the cross section increased in intensity, following the trend of the time mean secondary flow, whereas axial (streamwise) fluctuations became weaker. Overall turbulence levels decreased with the curvature and were lower in a curved pipe than in the straight pipe. Turbulence budgets over the cross section confirmed that in curved pipes production and other budget terms are comparable with those in the straight pipe (or even higher) in the outer bend region, whereas they rapidly decay as one moves towards the inner side. They also indicated that, in curved pipes, convection terms play a significant role in the turbulence budget, especially in the regions of the Dean vortices.

1. Introduction and literature review on flow in curved pipes

1.1. Problem definition and notation

Curved pipes are ubiquitous in engineering equipment involving fluid flow. For example, helical coils are widely used for heat exchangers and steam generators in power plant because they easily accommodate thermal expansion and may exhibit higher heat transfer rates than straight pipes [1–3]. Several studies, e.g. [4,5], show that coil torsion has only a higher order effect on the flow and on global quantities such as the friction coefficient. Therefore, significant insight into the basic mechanisms acting in coils can be derived by studies on curved pipes with their axis lying in a plane, which exhibit a smaller number of independent parameters.

Fig. 1 shows a schematic representation of a tract of a curved pipe; the radius of curvature will be indicated by c and the cross-section radius by a. The inner and outer sides will be indicated by "I" and "O" respectively; in the cross-section, the circumferential angle θ will be measured in the anti-clockwise direction looking from upstream, with $\theta(O) = 0$, $\theta(I) = \pi$.

In the following, the cross-sectional average of a generic quantity ϕ will be indicated by ϕ_{av} and its time average by $\bar{\phi}$ or Φ, while the fluctuation $\phi - \Phi$ will be indicated by ϕ'. The notation $\langle\phi\rangle$ will be used for the circumferential average of a wall quantity (e.g. the wall shear stress τ_w). The bulk Reynolds number Re will be defined on the basis of the pipe diameter $2a$ and the time- and cross-section-averaged (bulk) velocity U_{av} as $\mathrm{Re} = 2U_{av}a/\nu$, ν being the kinematic viscosity of the fluid. The friction-velocity Reynolds number will be defined as $\bar{u}_\tau^0 a/\nu$, where $\bar{u}_\tau^0 = \sqrt{\langle\bar{\tau}_w\rangle/\rho}$ is the friction velocity based on the time- and circumferentially-averaged wall shear stress. Finally, the curvature will be defined as $\delta = a/c$. Global wall scales can be built on the basis of $\bar{u}_\tau^0$, i.e. $\bar{u}_\tau^0$ itself for velocity, $\nu/\bar{u}_\tau^0$ for length, $(\bar{u}_\tau^0)^2$ for Reynolds stresses and turbulence energy, $\nu/(\bar{u}_\tau^0)^2$ for time, $(\bar{u}_\tau^0)^4/\nu$ for dissipation and other terms in the turbulence energy budget; the corresponding normalized quantities will be denoted by a superscript +. Similarly, local wall scales, corresponding to a given azimuth θ, can be built on the basis of

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Nomenclature

a	pipe radius (m)
c	curvature radius (m)
C_s	constant in Smagorinsky sub-grid model
De	Dean number, $\mathrm{Re}\sqrt{\delta}$
f	Darcy–Weisbach friction coefficient, $4\tau_w/(\rho u_{av}^2/2)$
f_μ	van Driest near-wall damping function, $1-\exp(-y^+/25)$
p	pressure (Pa)
p_s	streamwise driving force per unit volume (Pa m^{-1})
r	radial coordinate in the cross section (m)
Re	bulk Reynolds number
Re_τ	friction Reynolds number
S_{ij}	strain rate tensor, $(\partial u_i/\partial x_j+\partial u_j/\partial x_i)/2$
u	velocity (m s^{-1})
u_τ	friction velocity (m s^{-1})

Greek symbols

δ	curvature, a/c
Δ	average size of the computational mesh (m)
ε	dissipation (m^2 s^{-3})
ν	kinematic viscosity (m^2 s^{-1})
ν_s	sub-grid kinematic viscosity (m^2 s^{-1})
ρ	density (kg m^{-3})
Λ_K	Kolmogorov length scale (m)
θ	circumferential angle in the pipe's cross section (deg)
Θ	circumferential angle around the curvature axis (deg)
τ	shear stress (Pa)

Subscripts

av	cross sectional average
cr	critical
rms	root mean square
s, r, θ	streamwise (axial), radial and circumferential directions
w	wall

Superscripts

0	reference value
+	expressed in global wall units
*	expressed in local wall units

Averages and fluctuations

Φ_{av}	cross section-averaged
$\overline{\Phi}$	time-averaged
$\langle\Phi\rangle$	circumferentially-averaged
$\langle\overline{\Phi}\rangle$	time- and circumferentially-averaged
Φ'	fluctuating $(\Phi-\overline{\Phi})$

$\bar{u}_\tau=\sqrt{\bar{\tau}_w/\rho}$, and corresponding normalized quantities will be denoted by a superscript *.

The friction velocity $\bar{u}_\tau^0=\sqrt{\langle\bar{\tau}_w\rangle/\rho}$ can also be used to define the Large Eddy TurnOver Time (LETOT) $a/\bar{u}_\tau^0$, a time unit currently used in direct and large-eddy simulations of turbulence, e.g. to identify the minimum simulation time required for statistical significance. Since the wall time unit is $\nu/(\bar{u}_\tau^0)^2$, one LETOT corresponds to a number of wall time units equal to the friction velocity Reynolds number, $\bar{u}_\tau^0 a/\nu$, which is 500 in all cases discussed here.

1.2. Fundamental studies

Flow in curved pipes is characterized by the existence of a secondary circulation in the cross section, caused by the local imbalance between inertial and centrifugal forces [6]. The secondary flow appears when the Reynolds number exceeds $\sim 11.6/\delta^{1/2}$ [7], e.g. $\sim$36.7 for δ = 0.1 and $\sim$21.2 for δ = 0.3. The earliest studies of flow in curved pipes are due to Boussinesq [8], who identified the mechanisms driving the secondary flow in a curved duct and predicted the presence of two symmetric secondary vortices. Other early studies have been recently reviewed, for example, by Di Piazza and Ciofalo [9].

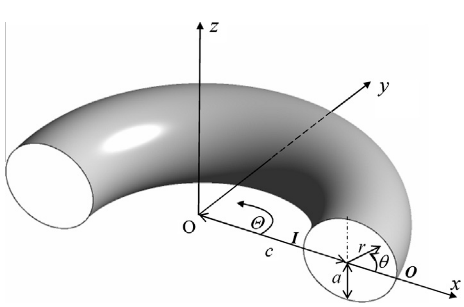

Fig. 1. Schematic representation of a curved pipe segment (computational domain) with its main geometrical parameters: a, tube radius; c, coil radius. The inner (I) and outer (O) sides of the curved duct are indicated; r and θ are local polar coordinates in the cross-section, with θ measured from "O" in the anti-clockwise direction (looking from upstream), while Θ is a global circumferential coordinate.

Dean [10] wrote the Navier–Stokes equations in a toroidal reference frame, and, under the assumptions of small curvature and laminar stationary flow, obtained power series solutions for the stream function of the secondary motion and for the main streamwise velocity; the first terms of the series give Hagen–Poiseuille flow, while higher-order terms account for the effects of curvature. From this analysis a new governing parameter emerged, now called the *Dean number* $\mathrm{De}=\mathrm{Re}\sqrt{\delta}$, which accounts for inertial, centrifugal and viscous effects. Although Dean originally based this parameter on the maximum velocity which would occur in a straight pipe under the given pressure gradient, most authors subsequently based De on the average streamwise velocity in the curved pipe, U_{av}; this latter convention will be adopted here.

1.3. Experimental work on transition to turbulence and friction

Through the years, curved and helical pipes have been the subject of a considerable amount of experimental studies, mainly motivated by their potential application in heat transfer equipment. Among the most significant contributions, we cite here the work by Ito [11] as concerns friction and by Mori and Nakayama [12,13] and Xin and Ebadian [14] as concerns heat transfer. State-of-the-art reviews of the subject have been given by Naphon and Wongwises [15] and by Vashisth et al. [16].

In the present paper, we are mainly interested in the conditions for transition to turbulence and in the hydrodynamic features of turbulent flow. As discussed e.g. by Di Piazza and Ciofalo [9], the effect of curvature is to increase the critical Reynolds number for transition to turbulence, Re_{cr}, with respect to straight pipes; an approximate correlation which synthesizes most of the published results is:

$$\mathrm{Re}_{cr}=2100\cdot(1+15\delta^{0.57}) \quad (1)$$

which, for example, provides Re_{cr} = 6368 for δ = 0.03 and Re_{cr} = 10,578 for δ = 0.1; the case δ = 0.3 is beyond the range of validity of Eq. (1).