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Single-mode bubble evolution simulations of Rayleigh Taylor instability with spectral element method and a viscous model ☆

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ABSTRACT

Based on the single-mode potential model of Jacobs and Rikanati, Firstly, a viscous single-mode bubble evolution model of Rayleigh Taylor instability (RTI) is developed in this study. Viscous effects of RTI's early stage growth for low Atwood number have been explained. In addition, direct numerical simulations of single mode RTI are studied with Navier–Stokes equations and a transport-diffusive equation for miscible fluids, in which these equations are discritized with discontinuous Galerkin (DG) spectral element method. The turbulent mixing of RTI has kinetic energy dissipation, and the dissipation rate is determined by the inertial and viscous effects. Therefore, the numerical techniques must include a dissipation mechanism for kinetic energy. For this reason, the high accurate spectral element method is employed in this study. Agreement between the theoretical model and the numerical results shows that simulations of RTI is feasible using the mathematical miscible fluid model. The results also suggest that a high order numerical method may provide the capability of simulating small scale fluctuations in turbulent flows with RTI.

1. Introduction

Rayleigh–Taylor instability (RTI) has been studied for many years because it plays an important role in many physical problems [1,2] such as high energy density physics (HEDP) [3] of magnetic [4] or inertial [5] fusion, interstellar gas [6], Earth's climate [7]. Still, researchers do not understand the basic physical mechanisms of RTI [1,2,8] very well and struggle to learn more about it. In terms of RTI study, single mode bubble evolution is still active as an important role for understanding its turbulent physics because a single mode RTI was thought to be the basis for the study of multi-mode RTI random perturbations [9,10,2,11]. There are some research works of RTI that mainly focused on the growth of a single mode perturbation either in the linear or nonlinear regimes [12–15], and these existing evolution models of RTI are mainly proposed based on potential flow theory. Layzer's potential flow model [16], which was successfully applied to two dimensional (2-D) planar and three-dimensional (3-D) cylindrical RTI bubbles, is widely accepted and has been validated or compared with experimental and numerical results. In his model, a linear regime means that the perturbation grows exponentially in time, followed by the appearance of bubbles and spike structures. The bubbles are in the form of columns of light fluid interleaved with falling spikes of heavy fluid. Eventually, the bubbles reach a constant velocity, while the spikes fall with a constant acceleration. The shortcomings of Layzers model are its limitation to incompressible fluids and its assumption of a unity Atwood number, $A = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$, though it has been shown over many years to serve as a useful guide for gaining understanding. Jacobs and Sheeley [17] proposed a vortex model based on his experimental results of Richtmyer–Meshkov instability, and the vortex model successfully predicted asymptotic bubble velocity. However, all these models neglect viscous effects, which may play an important role in RTI turbulent mixing.

Furthermore, due to the importance of RTI, various experimental techniques, theoretical analysis methods and numerical methods have been applied to its study since Taylor's first publication. In particular, numerical methods are the most popular techniques being used due to the rapid development of computational technologies. Previous attempts to simulate RTI turbulent mixing are mainly direct numerical simulations (DNS) [18] and large eddy simulations (LES). However, direct numerical simulation methods can only be used to study low Reynolds number fluid flows as we know, because the number of elements and the degree of the polynomial spaces have to be increased with increasing *Re*. Even the fastest supercomputers can hardly finish the small scale resolution requirements of high-Reynolds number turbulent flow simulations. Therefore, there have been some attempts in using LES simulations to reveal the physics of high-Reynolds number turbulent RTI mixing. Subgrid-scale (SGS) models of LES simulation assume a turbulent cascade of energy from large to small scales,

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Nomenclature

A	Atwood number
ρ	density
Υ	vortex strength
x, y	coordinates in physical domain
w	complex potential
ψ	stream function
λ	wave length of initial perturbation
u_2	vertical velocity scalar
a	interfacial amplitude
v	dynamic viscosity
V_0	the initial single mode perturbation velocity
t	time
Re	Reynolds number
D	diffusivity
L	the length of computational domain
g	gravity
Pr	Prantdl number
Fr	Froude number
Pe	Peclet number
p	pressure
Y	mass fraction of species
T	temperature
κ	thermal diffusion coefficient
τ	stress tensor
Q	state vector
F	flux vector
h	Lagrange interpolation function
ϕ	test function
ξ, η, ζ	coordinates in computation domain
θ	amplification factor
C	constant
σ	scalar function of filter
ϵ	error

however, RTI grows up from small scales. SGS models are not suitable for the early times RTI simulations. Due to the challenges associated with of RTI turbulence mixing numerical methods suited for studying RTI are still struggling for a better method.

The present study provided a novel viscous single-mode bubble evolution model of RTI that is an extension of the single-mode potential models of Jacobs and Sheeley [17] and Rikanati et al. [19]. Furthermore, a discontinuous Galerkin (DG) spectral element method suitable for miscible fluids is formulated, implemented and applied to RTI turbulent mixing simulation. The remainder of the paper is organized as follows. First, a viscous single-mode vortex evolution model is introduced. A short description of the mathematical model equations of RTI and a concise introduction of spectral element method are then presented. Third, the special treatment of interface discontinuity in spectral element method necessary to obtain a stable scheme and diminish Gibbs phenomenons is discussed in detail. Finally, the numerical results are presented and some conclusions are drawn.

2. A viscous single-mode bubble evolution model

In this study, numerical simulation results of our three-dimensional spectral element simulation for single-mode RTI have clearly shown the early formation of vortices in the flow patterns (see Fig. 1). This observation implies that the flow is rotational and can be described by vortex dynamics. Therefore, a viscous single-mode bubble evolution model is developed and described, which is an extension of the inviscid model developed by Jacobs and Sheeley [17] and Rikanati et al. [19].

In order to build this model, there are three assumptions: firstly, a linear assumption is used for the initial small amplitude phase, which allows for independent studies of potential and viscous phases that can then be added together [20]; secondly, the gravity potential can be neglected because the initial growth amplitude is small; finally, at initial small amplitude phase RTI is divided into two stages, initial vortex formation due to perturbation which can be described by potential flow solution, and diffusion process of vortex on the RTI interface.

An array of identical bubbles has been assumed for the single mode initial perturbations of RTI as shown in Fig. 2. Then, an initial vorticity field is generated due to the small amplitude of growth. Two vortices in opposite directions with a distance of half the wave length form a single bubble, in which λ is the perturbation wave length that represents double size of distance between neighboring vortices along the line. Based on complex potential theory, an array of bubbles can be modeled by a set of infinite vortex lines, and a single vortex is written as

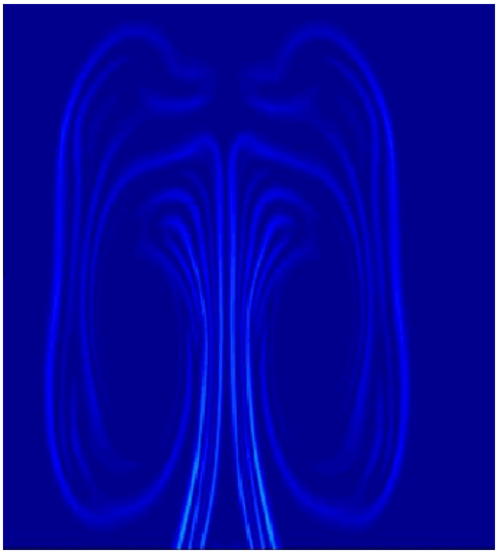

Fig. 1. The single-mode RTI bubble image from simulation results.

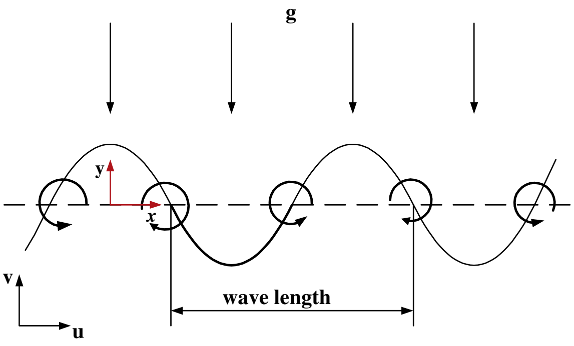

Fig. 2. Scheme of the single-mode vortex model at small amplitude phase.