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Simulations of flow instability in three dimensional deep cavities with multi relaxation time lattice Boltzmann method on graphic processing units

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ABSTRACT

The D3Q19 multi relaxation time (MRT) lattice Boltzmann model is adopted to simulate the instability phenomenon within a three-dimensional cavity at various depth–width aspect ratios ranging from 1 to 3. The computations are conducted on a single node multi graphic processing unit (GPU) system, consisting of three nVIDIA M2070 devices using OpenMP. Results show that transition takes place between $1750 < Re_{cr} < 1950$, for cubic cavity and $1100 < Re_{cr} < 1450$, for deep cavity flows with aspect ratio 2 and 3. This indicates that an increase of the depth–width aspect ratio would induce the transition at lower Reynolds number and is consistent with the previous results for two-dimensional cavities, though the critical Reynolds number is approximately 75% lower for three dimensional cavity.

1. Introduction

The lid driven cavity flow as a classical benchmark problem has been extensively studied using both numerical approaches and experimental techniques. Despite its geometric simplicity, the flow exhibits a variety of flow features and provides an ideal platform to examine the vortex dynamics. An overview of cavity flow related works can be found in Ref. [1].

Due to the difficulty to generating two dimensional cavity flow experimentally, most studies of two-dimensional cavity flows were investigated by numerical simulations. Pioneering works of Ghia et al. [2] and Schreiber and Keller [3] presented accurate simulations for 2-D square cavity flows, and these were thereafter considered as benchmark solutions. The square cavity flow has been vigorously examined by different numerical schemes [4–7]. Stability in such two dimensional systems is an issue of great interest and has been extensively studied. Albensoeder et al. [6] conducted both numerical and experimental studies to examine the linear stability of two-dimensional cavity flows with large spanwise length and concluded that the instability is fully three-dimensional and the stability properties are strongly dependent on the width-depth ratio. Also, numerical results revealed that the critical Reynolds number, where the first Hopf bifurcation takes place, is around 8000 for square cavity flow [8–10], while Lin et al. [7,11] further showed that this value decreases with the increase of the depth–width ratio.

On the other hand, three-dimensional cavity flows were investigated both by experiments and numerical simulations. Iwatsu et al. [12], Guj and Stella [13] and Mei et al. [14] adopted different numerical schemes to simulate cubic cavity flows, where steady solution was shown to exist at Reynolds number being $Re = 2000$. Using Chebyshev-collocation technique, Albensoeder and Kuhlmann [15] presented solutions for cavity flows at various aspect ratio at Reynolds number up to 1000. Feldman and Gelfgat [16] also investigated numerically the critical Reynolds number for cubic cavity and showed that the oscillatory instability occurs at $Re \approx 1914$. This result was later supported by a PIV measurement [17], which concluded that the critical Reynolds number locates in the region between $1700 < Re_{cr} < 1970$.

Previous works indicated that aspect ratio has strong influence on the fluid stability [6,11]. However, this topic has received little attention. Therefore, the present study aims to examine the range of critical Reynolds number as well as the relationship between critical Reynolds number Re_{cr} and depth–width ratio. The D3Q19 MRT lattice Boltzmann model is adopted here due to its enhanced stability at high Reynolds number flows.

As an explicit numerical scheme with intensive local computation, the LBM algorithm is very suitable for parallelization. This can be achieved using the graphical processing unit (GPU) through the Compute Unified Device Architecture (CUDA). Single graphic processing unit has been successfully used for lattice Bolztmann

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computations [18,19,11], and some [20,21] further extended to multiple GPUs platform. Due to the immense computing demand for three dimensional MRT lattice Bolztamnn simulation, multi-GPU computations will also be adopted. The computation platform is a single node multi-GPU system consisting of three nVIDIA M2070 devices with OpenMP framework and its performance relative to CPU will also be addressed.

2. Multi relaxation time lattice boltzmann model and boundary conditions

The multi relaxation time (MRT) lattice Boltzmann method [22] can be expressed by collision and streaming steps, respectively as:

$$f_i^*(\vec{x},t) = f_i(\vec{x},t) - M_{il}^{-1} S_{lj}[m_j(\vec{x},t) - m_j^{eq}(\vec{x},t)] \quad (1)$$

$$f_i(\vec{x} + \vec{e}_i \triangle t, t + \triangle t) = f_i^*(\vec{x},t) \quad (2)$$

where **M** is a matrix that transforms the distribution function **f** to the velocity moment, **m** = *Mf*, and **S** is the relaxation matrix. These will be defined later.

Based on the particle distribution functions, the macroscopic density and velocity can be obtained as:

$$\sum_i f_i = \rho, \sum_i f_i \vec{e}_i = \rho \vec{u} \quad (3)$$

For the present 3D applications, the D3Q19 multi relaxation time model is adopted. The transform matrix **M** of this model is given as [23],

$$M = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-30 & -11 & -11 & -11 & -11 & -11 & -11 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \\
12 & -4 & -4 & -4 & -4 & -4 & -4 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & -4 & 4 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\
0 & 0 & 0 & -4 & 4 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & -4 & 4 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
0 & 2 & 2 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -2 & -2 & -2 & -2 \\
0 & -4 & -4 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -2 & -2 & -2 & -2 \\
0 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2 & -2 & 2 & 2 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1
\end{bmatrix} \quad (4)$$

and the velocity moments are correspondingly defined as,

$$\mathbf{m} = (\rho, e, \varepsilon, j_x, q_x, j_y, q_y, j_z, q_z, 3p_{xx}, 3\pi_{xx}, p_{ww}, \pi_{ww}, p_{xy}, p_{yz}, p_{zx}, m_x, m_y, m_z)^T \quad (5)$$

As suggested in [23], the adopted equilibrium moments are,

$$\begin{aligned}
&e^{eq} = -11\rho + \frac{19}{\rho}\left(j_x^2 + j_y^2 + j_z^2\right), \quad \varepsilon^{eq} = -\frac{475}{63}\frac{1}{\rho}\left(j_x^2 + j_y^2 + j_z^2\right) \\
&q_x^{eq} = -\frac{2}{3}j_x, \quad q_y^{eq} = -\frac{2}{3}j_y, \quad q_z^{eq} = -\frac{2}{3}j_z \\
&3p_{xx}^{eq} = \frac{1}{\rho}\left[2j_x^2 - \left(j_y^2 + j_z^2\right)\right], \quad p_{ww}^{eq} = \frac{1}{\rho}\left(j_y^2 - j_z^2\right) \\
&p_{xy}^{eq} = \frac{1}{\rho}j_x j_y, \quad p_{yz}^{eq} = \frac{1}{\rho}j_y j_z, \quad p_{xz}^{eq} = \frac{1}{\rho}j_x j_z \\
&3\pi_{xx}^{eq} = \pi_{ww}^{eq} = 0, \quad m_x^{eq} = m_y^{eq} = m_z^{eq} = 0
\end{aligned} \quad (6)$$

Here, the relaxation matrix **S** is a diagonal matrix, i.e.,

$$S = diag[0, s_1, s_2, 0, s_4, 0, s_4, 0, s_4, s_9, s_{10}, s_9, s_{10}, s_{13}, s_{13}, s_{13}, s_{16}, s_{16}, s_{16}] \quad (7)$$

where the kinematic viscosity is given by

$$\nu = \frac{1}{3}\left(\frac{1}{s_9} - \frac{1}{2}\right) = \frac{1}{3}\left(\frac{1}{s_{13}} - \frac{1}{2}\right) \quad (8)$$

and $s_9 = s_{13}$ are determined based on the kinematic viscosity. The relaxation parameters for density and momentum are set equal to zero in order to conserve mass and momentum. For the present three dimensional cavity, it was found that convergent solution for cubic cavity at high Reynolds number, such as *Re* = 7500, can only be obtained if the non-viscosity related relaxation parameter is 0.7. It is also found that at lower Reynolds number flow, this does not affect the solution accuracy. Thus, in subsequent computations, the non-viscosity related relaxation parameter is set to be 0.7.

For the present lid driven cavity shown in Fig. 1, two types of boundary conditions are adopted. The first one is used for the top lid boundary which moves at a constant velocity, while the second one is applied on the stationary boundary along the remaining five walls. Boundary conditions proposed in [24,25] are employed to determine the unknown particle density distribution functions along the boundary, which are expressed as a combination of the local known value and a corrector,