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Dynamics of pairs and triplets of particles in a viscoelastic fluid flowing in a cylindrical channel

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ABSTRACT

The dynamics of pairs and triplets of particles suspended in a viscoelastic fluid flowing along the centerline of a cylindrical channel is studied by numerical simulations. The governing equations are solved by the finite element method by employing an ALE formulation to handle the particle motion.

For a pair of particles, at variance with the Newtonian case, the viscoelastic nature of the suspending medium alters the interparticle distance during the flow. For low and moderate Deborah numbers, the particles can approach or separate depending on the initial distance. For high Deborah numbers, the approaching dynamics disappears. Different fluid rheology and confinement ratio only quantitatively alter such a scenario.

The three-particle dynamics is more complex. In a Newtonian liquid, the leftmost particle of the triplet approaches and slows down the middle one. Consequently, the rightmost particle separates, giving rise to a pair and an isolated particle. A similar scenario occurs for a viscoelastic liquid. In this case, however, depending on the initial configuration and the Deborah number, the particles forming the pair can subsequently approach or separate. In the latter case, the final configuration is the formation of three isolated particles.

1. Introduction

The motion of suspensions in pressure-driven flows occur in several fields. It is well-known that a number of factors, such as inertia [1,2], viscoelasticity [3–5], high solid concentration [6–8], Brownian motion [9,10], may lead to inhomogeneous particle distributions. For instance, in dilute suspensions flowing in a cylindrical channel, inertial effects drive the particles towards an equilibrium annulus [1]. In contrast, fluid viscoelasticity (in inertialess conditions) promotes the migration towards the channel centerline and, for remarkably shear-thinning fluids, also towards the wall (inversion of Segré–Silberberg effect) [11]. As the particle volume fraction increases, particle–particle hydrodynamic interactions arise, that may strongly alter the particle distribution along the channel.

The dynamics of rigid particles and drops flowing in a Newtonian inertialess fluid between two parallel plates at high confinement ratios has been recently investigated by Janssen et al. [12]. Limiting to rigid particles, the authors showed by numerical simulations that linear arrays of spheres aligned in the flow direction exhibit a particle-pairing instability leading to the formation of isolated pairs at increasing distances along the flow.

For Newtonian suspending liquids at finite Reynolds numbers, Matas et al. [2] experimentally reported the formation of particle trains in tube flows. The particles were observed to migrate towards the Segré–Silberberg equilibrium annulus and form aligned structures at sufficiently high Reynolds numbers. More recently, particle trains have been investigated in rectangular microfluidic devices [13,14]. Particle pairs may show complex dynamics characterized, for instance, by damped oscillations, i.e. increasing and decreasing interparticle distances, before achieving a steady-state regime [13]. The formation of self-assembled trains follows similar dynamics as the pair and inertia is found to stabilize the interparticle spacing [13]. In Humpry et al. [14], the formation of multiple parallel chains aligned along the flow direction is investigated. The number of trains is found to depend on channel cross-section and particle volume fraction. Numerical simulations showed that, at small but finite Reynolds numbers, the confinement leads to two inward spiraling regions forth and back a flowing particle, defining the positions of neighboring particles in the axial direction [14]. Therefore, the appearance of multiple particle trains is an hydrodynamic interaction effect related to fluid inertia.

In non-Newtonian fluids with negligible inertia, structure formation has been widely studied in shear flows [15–17]. As recently reported for confined systems, the migration mechanism, driving

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the particles towards the walls, competes with the formation of structures aligned along the flow direction in the bulk [17].

To the best of our knowledge, studies on particle structure formation in viscoelastic fluids in pressure-driven flows are not available. We point out that, in tube flows, fluid viscoelasticity drives the (most of the) flowing particles towards the channel centerline, i.e. a flow-focusing mechanism occurs [11]. Therefore, at variance with shear flows, the formation of structures aligned along the channel centerline is promoted by the viscoelasticity-induced migration effect. It is worthwhile to remark that, even for dilute systems, once trains flowing along one streamline are formed, particle–particle hydrodynamic interactions are expected to arise because of the relatively small interparticle distances. Hence, the focused particles do not behave as isolated objects but peculiar dynamics due to fluid viscoelasticity may occur.

In this regard, we mention the widely studied problem of two spheres settling along the line connecting their centers as a typical example where hydrodynamic interactions mediated by fluid viscoelasticity play an important role, leading to different dynamics depending on fluid rheology. For instance, Riddle et al. [18] experimentally investigated the effect of the distance between two spherical particles falling along their line of centers in shear-thinning viscoelastic fluids. They found that the spheres approach for small initial separation distances and repel for large distances, i.e. an unstable critical separatrix exists. In contrast, the experiments of Bot et al. [19] in a Boger suspending liquid showed that the spheres attract for large distances but separate for small distances (stable critical separation distance). An attractive force when the free stream is along the line of centers was also reported by Feng et al. [20] for an Oldroyd-B suspending liquid and by the recent analysis of Ardekani et al. [21] for a Second Order Fluid.

Of course, the two-sphere falling problem is quite different from the pressure-driven flow of particles in channels where the motion is generated by a flow rate/pressure difference (instead of a constant force as the gravity). Nevertheless, the results available in the literature clearly show the strong effect of the fluid rheology on particle dynamics.

In this work, we perform numerical simulations to study the dynamics of pairs and triplets of spherical particles suspended in a viscoelastic inertialess liquid flowing in a pipe. We assume that the flow-focusing mechanism already occurred, thus the particles are aligned along the channel centerline. The particles are considered spherical with equal radius, rigid, non-Brownian and inertia is neglected. The liquid is modelled by the Giesekus constitutive equation.

To understand the fundamental aspects of the effect of viscoelasticity on the particle–particle hydrodynamic interactions, the dynamics of a particle pair is firstly investigated by varying the initial interparticle distance, the flow intensity, the fluid rheology and the confinement ratio. Based on the pair particle results, the study is extended to a three-particle system. The phase diagram reporting the evolution of the relative particle distances is shown for different Deborah numbers, highlighting different regimes and long-time configurations. Finally, the conclusions are summarized and future work is discussed.

2. Mathematical model

2.1. Governing equations

The system considered in this work consists in a cylindrical channel filled by a viscoelastic fluid with two (pairs) or three (triplets) flowing particles. In Fig. 1a, the case of three particles is sketched. The particles are considered rigid, non-Brownian and spherical, with the same diameter $D_p = 2R_p$, and are assumed to flow along the channel centerline (i.e. after the focusing mechanism). The main flow direction is along the z-axis. The particles are progressively numbered from the left to the right, as reported in Fig. 1b. We denote by L and $D_c = 2R_c$ the length and the diameter of the channel, respectively. The inflow and outflow sections are denoted by $\partial\Omega_{in}$ and $\partial\Omega_{out}$. A flow rate Q is set on the inflow section, and periodic conditions are imposed between the inflow and outflow sections. Provided that the length L of the domain is sufficiently larger than the particle diameters, hydrodynamic interactions between the leftmost and rightmost particles through the periodic boundaries can be neglected.

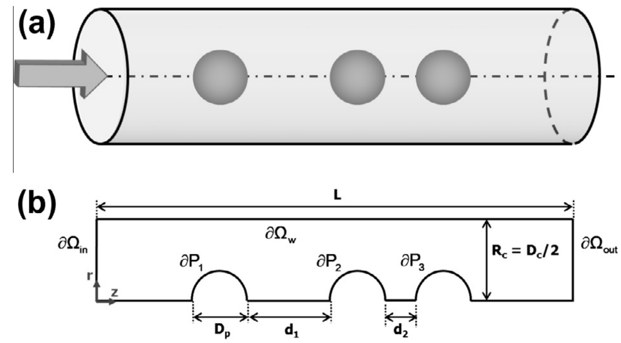

Fig. 1. (a) Scheme of the problem investigated in this work. (b) Computational domain used in the simulations and relative notations. In both figures the general case of three particles flowing at the channel centerline are reported.

Because of the rotational symmetry, the particles stay on the channel centerline and do not rotate. The positions of the particle center along the z-axis are denoted by z_i with $i = 1,2$ for pairs and $i = 1,2,3$ for triplets. The only relevant kinematic quantity is the z-component of the translational velocity U_i, being both the radial and angular components nil.

Neglecting both fluid and particle inertia, the governing equations for the fluid motion are the continuity and the momentum equations:

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$\nabla \cdot \boldsymbol{\sigma} = \boldsymbol{0} \tag{2}$$

where $\boldsymbol{u}$ is the fluid velocity and $\boldsymbol{\sigma}$ is the total stress tensor, expressed as:

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\eta_s\boldsymbol{D} + \boldsymbol{\tau} \tag{3}$$

In Eq. (3) p, $\boldsymbol{I}$, η_s and $\boldsymbol{D} = (\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T)/2$ are the pressure, the unity tensor, the viscosity of a Newtonian 'solvent' and the rate-of-deformation tensor, respectively. Finally, $\boldsymbol{\tau}$ is the viscoelastic stress tensor that needs to be specified by choosing a constitutive equation. In this work, we consider the Giesekus model:

$$\lambda \overset{\triangledown}{\boldsymbol{\tau}} + \frac{\alpha\lambda}{\eta_p}\boldsymbol{\tau}\cdot\boldsymbol{\tau} + \boldsymbol{\tau} = 2\eta_p\boldsymbol{D} \tag{4}$$

where η_p is a viscosity, λ is the fluid relaxation time, the symbol ($^\triangledown$) denotes the upper-convected time derivative:

$$\overset{\triangledown}{\boldsymbol{\tau}} \equiv \frac{\partial\boldsymbol{\tau}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{\tau} - (\nabla\boldsymbol{u})^T\cdot\boldsymbol{\tau} - \boldsymbol{\tau}\cdot\nabla\boldsymbol{u} \tag{5}$$

and α is the (dimensionless) constitutive parameter. We recall that the Giesekus model predicts shear-thinning for the constitutive parameter α greater than zero, and non-zero first and second normal stress differences. A limiting case is $\alpha = 0$, corresponding to the Oldroyd-B model.

Regarding the boundary conditions, no-slip and rigid-body motion are imposed at the sphere surfaces: