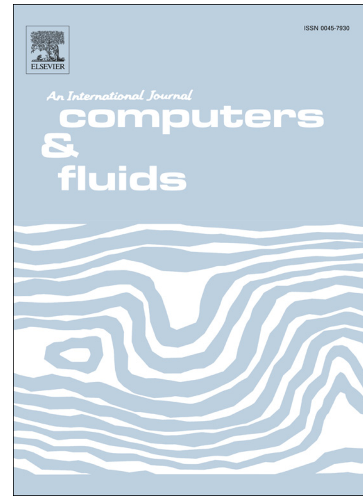
Accepted Manuscript



Construction of exact solutions for fractional-type difference-differential equations via symbolic computation

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PII: S0045-7930(13)00279-X
DOI: http://dx.doi.org/10.1016/j.compfluid.2013.07.008
Reference: CAF 2239

To appear in: *Computers & Fluids*

Received Date: 5 June 2013
Revised Date: 8 July 2013
Accepted Date: 10 July 2013

Please cite this article as: Aslan, İ., Construction of exact solutions for fractional-type difference-differential equations via symbolic computation, *Computers & Fluids* (2013), doi: http://dx.doi.org/10.1016/j.compfluid.2013.07.008

Construction of exact solutions for fractional-type difference-differential equations via symbolic computation

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Abstract

This paper deals with fractional-type difference-differential equations by means of the extended simplest equation method. First, an equation related to the discrete KdV equation is considered. Second, a system related to the well-known self-dual network equations through a real discrete Miura transformation is analyzed. As a consequence, three types of exact solutions (with the aid of symbolic computation) emerged; hyperbolic, trigonometric and rational which have not been reported before. Our results could be used as a starting point for numerical procedures as well.

Keywords: Difference-differential equation; lattice equation; extended simplest equation method

1. Introduction

The appearance of difference-differential equations (DDEs) or lattice equations in nature is quite common. They model many physical and engineering problems such as [illegible], pulses in biological chains, currents in electrical networks, particle vibrations in lattices, [illegible], [illegible], etc. Their important role has motivated investigators to develop a number of integrable DDEs since the original work of Fermi, Pasta and Ulam [1]. To name a few; Volterra lattice equation [2], discrete KdV equation [3], Toda lattice equation [4], Ablowitz-Ladik lattice equation [5], discrete sine-Gordon equation [6], discrete modified KdV equation [7], see [8] for a list of DDEs. These DDEs are of (or can be converted to) the form $\dot{u}_n = P(\ldots, u_{n-1}, u_n, u_{n+1}, \ldots)$ where P is a polynomial of its arguments. Unlike difference-difference equations which are completely discretized, DDEs are semi-discretized with some (or all) of their space variables discretized while time is usually kept continuous. For this reason, they can be thought as hybrid equations. Apart from their physical relevance, DDEs also play a crucial role in numerical simulations of nonlinear partial differential equations.

In this study, our attention is focused towards fractional-type DDEs of the form

$$\dot{u}_n = R(\ldots, u_{n-1}, u_n, u_{n+1}, \ldots) \tag{1}$$

where $u_n(t) = u(n,t)$ the displacement of the nth particle from the equilibrium position and $n \in \mathbb{Z}$. Eq. (1) is called fractional-type in the sense that R is a rational function of its arguments. First, we consider the integrable equation [9, 10]

$$\dot{u}_n = \frac{u_{n-1} - u_{n+1}}{1 + u_{n-1} - u_{n+1}}, \tag{2}$$

from which the discrete KdV equation can be directly produced [11]. Second, our target will be the system [12]