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No-slip wall acoustic boundary condition treatment in the incompressible limit

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ABSTRACT

A characteristic formulation for the numerical treatment of acoustically reflecting no-slip wall boundary condition is presented and numerically validated for some discriminating situations. As an extension of the 3DNSCBC popular approach, this NSWIL strategy relaxes smoothly towards a 3DNSCBC strategy for a slipping wall – the Euler equations natural wall boundary condition – when the viscosity goes to zero. Using our in-house 6th order FD solver, some comparative tests were performed. In particular, we computed a pressure wave train in a 2D periodic channel, leading to standing acoustic waves. Long time runs using NSWIL strategy and involving 2.5×10^5 temporal iterations and 2×10^3 acoustic reflections at the walls show no numerical instability while popular NSCBC strategy turns out to be unstable after less than 100 reflections. In that case, global mass conservation was very precisely ensured using NSWIL (relative loss $<6 \times 10^{-5}$ after 2000 acoustic reflections) while NSCBC induced a global variation above 1% before code crashed.

1. Introduction

Accurate and stable no-slip wall boundary condition for compressible flows is still an open problem for general high-order finite difference solvers. As this will be pointed out in the paper, for the compressible high-order finite differences simulations we performed, we observed that original characteristic-based [17] no-slip wall boundary condition frequently leads to numerical instabilities. Indeed, as outlined in [21], when the order of the difference equations is higher than that of the Euler equations, the zero normal velocity boundary condition is insufficient to define a unique solution. Some strategies were proposed in the literature to use both high-order schemes and no-slip wall conditions when solving fully compressible NS equations. Some of these methods are based on the use of "ghost cells" [5]. Initially, they were developed by Tam and Dong [21] then extended to curvilinear cases by Hixon [8]. The idea is to add external points to the physical domain. For finite difference schemes, the coefficients assigned to the variables (velocity components, density, pressure) associated with these points are such that the no-slip wall boundary condition is ensured while advancing in time. More recently, Svärd and Nordström [20] proposed to directly modify the discretization scheme in order to stabilize the whole scheme. They use high-order accurate finite difference summation-by-parts operators. The boundary conditions are imposed weakly making use of penalty terms. All these proposed stabilizing solutions rely on a local modification of the discretization scheme.

The NSCBC strategy [17] is a popular technique to numerically handle compressible (acoustic) boundary condition. It proposes a way to make a distinction between incoming and outcoming waves. Based on this distinction, a specific treatment is proposed at the computational domain boundaries: outcoming waves are directly calculated whereas incoming waves would be modelled, that we call the "modelling stage" of the strategy. Once waves variations amplitudes are estimated, they are used to rebuild the *numerical* Navier Stokes (NS) $\frac{\partial}{\partial x_n}$ operator. Because at the boundaries a one-sided scheme is unable to pertinently describe information coming from the "outside" (e.g. as a wave reflection), to correctly estimate the operator $\partial./\partial x_n$ – with x_n the coordinate normal to the wall – BC treatment requires a specific strategy. At the "modelling stage", the initial NSCBC treatment assumed the flow to be locally one-dimensional, along the normal to the boundary, and inviscid: the so-called LODI, for Locally One-Dimensional Inviscid, assumption. Recent studies show that taking into account the three-dimensionality of the flow at the boundary yields far better behaviours for inflow or outflow BC treatment [23,14,4]. The LODI assumption is there replaced by a L3DI assumption.

These 3DNSCBC strategies have been successfully implemented for inflows and outflows. For wall boundary conditions however, it has been assumed that, as a consequence of the no-slip, there is no

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difference considering the LODI or L3DI assumptions [13]. It is that point that we shall mainly discuss in the present article. For no-slip walls, a contradiction inherently appears when making use of 3DNSCBC. More specifically, in [17], it is stated that: *"The NSCBC method is valid for the Navier–Stokes and Euler equations and relaxes smoothly from one to the other when the viscosity goes to zero"*. If the LODI assumption is made then the previous statement is valid but, if we consider the L3DI assumption, this is more questionable. Close to a no-slip wall, viscous effects may be important but should not prevent acoustic wave reflections. In presence of viscosity, considering the multi-dimensionality (L3DI) of the flow at the wall will lead to a different behaviour than if we consider that the flow is locally unidimensional (LODI).

In the present paper, we propose and implement a numerically stable[2] no-slip isothermal wall boundary condition in a 6th order finite-difference solver, with hybrid staggered/colocated structured grid [2,1], and with no modification of the discretization scheme, to enable dealing with consistent and convenient boundary conditions. Close to the boundaries, since the size of the stencil decreases, the scheme order is successively lowered to centred 4th order and then one-sided 3rd order [2].

The idea of the present proposed approach is as follows. Close to a boundary, the original NSCBC formulation makes use of a LODI assumption: transverse and diffusive terms are omitted at the boundary. For a slipping wall, taking into account the three-dimensionality of the flow seems however a better suited strategy, closer to real flow conditions. This will include transverse terms in the BC treatment and leads to a L3DI (for Locally Three-Dimensional Inviscid) assumption instead of a LODI assumption. Hence, close to the slipping wall, the characteristic formulation of the Euler equations (see system (12) in the sequel) should include transverse terms. On the other hand, the asymptotic limit of these Euler equations close to a wall is the incompressible Euler equations (see [18]). We shall consider that the no-slip wall is the limit of the (3D) slipping wall when the tangential velocity components go to zero – because of the viscous effect – i.e. is a particular case of the slipping wall.

For an isothermal no-slip wall, we would like to make use of a characteristic BC strategy, impose zero velocity and constant temperature, and, at the same time, be able to satisfy this physical incompressible wall limit of the (3D) Euler equations. For this purpose, we use a 3DNSCBC approach – well–suited for aero-acoustics, or compressible reactive flows – and modify it in order to fulfill the incompressible limit of the Euler equations close to the isothermal no-slip wall.

The present proposed approach will be referred to in the sequel as 3DNSCBC-NSWIL, for Three-Dimensional Navier–Stokes Characterictic Boundary Condition for No Slip Walls in the Incompressible Limit, or NSWIL for short. Skipping the normal velocity gradient in the NSCBC formulation this NSWIL approach proves to provide both formal high accuracy AND practical numerical stability. As shown by the presented numerical tests, it is able to pertinently describe acoustic reflection at the wall and numerically ensure global mass conservation, at least in all the numerical tests we performed. The analysis in [11] compares Dirichlet and NSCBC approaches in the case of a slipping adiabatic wall. It demonstrates that, in this case, a much better – more stable – description is obtained when using NSCBC instead of Dirichlet treatment. We will also compare our method to the direct Dirichlet method and use the result of Ref. [11] to explain why our method appears as more stable in practice.

The paper is organized as follows. Section 2 presents the finite-difference solver used for our test computations, namely the

[2] At least in all the numerical tests performed; note that we did not show analytical stability, see Section 3.

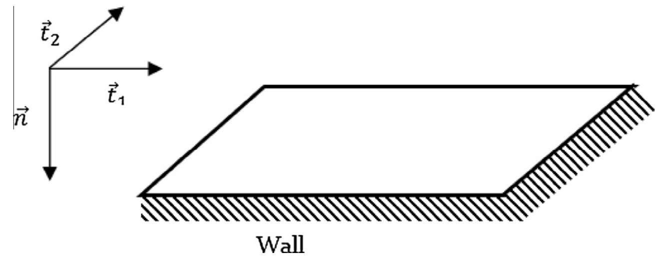

Fig. 1. Sketch of vector basis used for wall BC formulation.

in-house solver HAllegro, as well as the usual formulations (Dirichlet and characteristic) for wall treatment, that will be compared to the present proposed approach. Section 3 arises the inherent contradiction of using a characteristic formulation coupled to a no-slip condition and presents the no-slip wall treatment NSWIL. Numerical validations assess the practical effectiveness of the proposed strategy, as it is shown in Section 4. Concluding remarks end the paper in Section 5.

2. Wall boundary conditions for NS equations

The unit vectors of the orthonormal basis used to described the equations at the wall are denoted $(\mathbf{n}, \mathbf{t}_1, \mathbf{t}_2)$, with $\mathbf{n}$ the normal exterior vector and $\mathbf{t}_1$ and $\mathbf{t}_2$ the tangential vectors to the wall, see Fig. 1. Corresponding subscripts for vector coordinates will respectively be denoted n, t_1 t_2, or simply t to denote any of both tangential directions.

2.1. Governing equations and FD solver

The code HAllegro is an in-house finite-difference tool that has been developed for parallel Direct Numerical Simulation of the fully compressible subsonic reactive Navier–Stokes equations on structured meshes. For more details and capabilities of the code, we refer to [1,2,4,3]. The equations solved by the code are the multi-component Navier–Stokes equations for a (possibly reactive) compressible viscous flow. In cartesian coordinates, with Einstein summation convention, the system of non-reactive NS equations reads, in N space dimensions and in conservative form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho U_j}{\partial x_j} = 0, \tag{1a}$$

$$\frac{\partial \rho U_i}{\partial t} + \frac{\partial \rho U_i U_j}{\partial x_j} + \frac{\partial P}{\partial x_i} = \frac{\partial \tau_{ij}}{\partial x_j}, \tag{1b}$$

$$\frac{\partial \rho E}{\partial t} + \frac{\partial (P + \rho E) U_j}{\partial x_j} = \frac{\partial q_E^j}{\partial x_j} + \frac{\partial \tau_{ij} U_i}{\partial x_j} + S_E, \tag{1c}$$

Density is ρ, P denotes pressure, ρU_i are momentum components (for $i = 1, \ldots, N$). To close the system, using standard notations, it is assumed that $\tau_{ij} = \mu(\partial U_i/\partial x_j + \partial U_j/\partial x_i) - (2/3)\mu(\partial U_k/\partial x_k)\delta_{ij}$; $\mu = \mu_o (T/T_o)^{0.76}$; $\rho E = \rho C_V T + \rho U_i^2/2$; $q_E^j = \lambda \partial T/\partial x_j$; $\lambda = \mu C_p/Pr$ and $D = \mu/Sc_k$. Pressure is computed from perfect gas law $P = \rho r T$ (notice that specific perfect gas constant r – and hence specific heats $c_v = \frac{r}{\gamma - 1}$ and $c_p = c_v + r$ – are assumed constant). We added the source term S_E to include a heat release term in the energy equation. This will be used in the pressure train test-case in Section 4.4.

2.2. Finite-difference solver

The code is based on a compact 6th order finite-difference explicit scheme on hybrid structured grids. Time stepping makes