Computers & Fluids 86 (2013) 109–124

Contents lists available at ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

A physically consistent weakly compressible high-resolution approach to underresolved simulations of incompressible flows

Felix S. Schranner *, Xiangyu Y. Hu, Nikolaus A. Adams

Technische Universität München, Institute of Aerodynamics, 85747 Garching, Germany

ARTICLE INFO

Article history:
Received 25 February 2013
Received in revised form 24 June 2013
Accepted 28 June 2013
Available online 15 July 2013

Keywords:
Weakly compressible model
High resolution scheme
Implicit large-eddy simulation
Physically consistent
Underresolved computation

ABSTRACT

In engineering applications critical complex unsteady flows often are, at least in certain flow areas, only marginally resolved. Within these areas, the truncation error of the underlying difference schemes strongly affects the solution. Therefore, a significant gain in computational efficiency is possible if the truncation error functions as physically consistent, i.e. reproducing the correct evolution of resolved scales, subgrid-scale (SGS) model. The truncation error of high-order WENO-based schemes can be exploited to function as an implicit subgrid-scale (SGS) model. A recently developed sixth-order adaptive central-upwind weighted essentially non-oscillatory scheme with implicit scale-separation has been demonstrated to incorporate a physically consistent implicit SGS model for compressible turbulent flows. We consider the implicit SGS modeling capabilities of an improved version of this scheme simultaneously for underresolved turbulent and non-turbulent incompressible flows, thus extending previous works on this subject to a more general scope. With this model we are able to reach very long integration times for the incompressible Taylor–Green vortex at infinite Reynolds number, and recover in particular a low-mode transition to isotropy. Inviscid shear-layer instabilities are resolved to highly nonlinear stages, which is shown by considering the doubly periodic two-dimensional shear layer as test configuration. Proper resolved-scale prediction is also obtained for viscous–inviscid interactions and fully confined viscous flows. These properties are demonstrated by applying the model to a vortex–wall interaction problem and lid-driven cavity flow.

1. Introduction

Truncation error asymptotic analysis (sufficiently small grid spacing) is hardly relevant for practical applications, when the available grid resolution in certain parts of the computational domain is far from resolving all physically relevant flow structures. Thus, in most practical computations, the effect of the truncation error is not small and it contributes to the solution as an effective subgrid-scale model. The idea arises to adjust the local truncation error in order to function as a physically consistent subgrid-scale (SGS) model, i.e. delivering an accurate solution for resolved slow structures without determinating its asymptotic behavior at the fine-resolution limit. Modified differential equation analysis (MDEA) [1] has enabled us to show that the truncation error of nonlinear discretization schemes can be constructed such as to represent an implicit SGS model for turbulent flows [2]. It is known that the nonlinear regularization mechanism of high-order finite-volume schemes with shock-capturing capabilities can be used for implicit large eddy simulations (LES), for a review refer to [3]. A spectral extension of MDEA has allowed for designing the truncation error of a nonlinear scheme such that it recovers the theoretical spectral eddy viscosity when the flow is turbulent and underresolved. Such a situation, where the non-negligible local truncation error of a numerical scheme recovers correct physical SGS behavior is called in the following "physically consistent" behavior [4,5] in order to distinguish the analysis from that for asymptotically small truncation errors. Successful applications for physically consistent implicit LES models have been shown for a wide range of compressible and incompressible turbulent flows, e.g. [6–8]. Hu and Adams [9] have investigated the physical consistency of the underresolved contribution of an existing low-dissipation scheme (WENO-CU6) [10]. A proper modification of WENO-weights has resulted in a scale separation between contributions from the resolved and non-resolved scales to the locally reconstructed solution so that non-resolved scales are subject to dissipation, while the shock-capturing capabilities and the sixth order of accuracy in smooth flow regions of the underlying scheme are maintained.

Shu et al. have studied the evolution of the nearly incompressible, inviscid three-dimensional Taylor–Green vortex (TGV) [11]. They have found that the fifth-order WENO scheme shows unphysical dissipation effects but allows for stable underresolved

* Corresponding author. Tel.: +49 89 289 16396; fax: +49 89 289 16139.
E-mail addresses: felix.schranner@aer.mw.tum.de (F.S. Schranner), xiangyu.hu@tum.de (X.Y. Hu), Nikolaus.Adams@tum.de (N.A. Adams).

0045-7930/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compfluid.2013.06.034

simulations. It is well established that spectral methods are most efficient in well-resolved cases but cannot provide SGS energy transfer without explicit SGS models, e.g. [12,11]. In [11] it has been demonstrated that standard finite-difference and spectral methods do not provide the basis for physically consistent implicit SGS modeling capability. Furthermore, it has been found that integral flow quantities, such as enstrophy and kinetic energy, do not allow for a clear assessment of underresolved flow simulations. On the other hand, a discretization scheme that reproduces self-similar Kolmogorov spectra of a decaying isotropic turbulent flow at infinite Reynolds numbers is more likely to be robust to under-resolution. As WENO weighting involves measuring of flow resolution, it offers the potential to derive a physically consistent high-resolution scheme with a truncation error adjusted such that it exhibits implicit subgrid-scale modeling capabilities for both, turbulent and non-turbulent flows. We emphasize that for extremely large-scale simulations on massively parallel computers the weakly compressible flow model faces renewed significance as an alternative to strictly incompressible approaches. This is due to the fact that the weakly compressible flow model inherently requires less memory communication, as all operations are local unlike the strictly incompressible model, where the elliptic pressure-projection leads to global communication needs.

The objective of the current paper is to develop and to investigate physical consistency of a weakly compressible non-linear high-resolution approach for the under-resolved simulation of turbulent and non-turbulent incompressible flows. Due to the lack of analytic accessibility of the case dependent large truncation errors that occur in these cases, such an analysis mostly needs to rely on empirical investigation for a range of carefully selected test flow configurations that capture the essential properties of later target applications. A conservative approximate Roe–Pike solver is adapted to a weakly compressible flow model and combined with a low-dissipation WENO scheme. A modification of the underlying WENO scheme is proposed in order to obtain physical consistency of the resulting implicit SGS model.

As reference flow for implicit SGS model development we consider the three-dimensional Taylor–Green vortex (TGV) at infinite Reynolds number, in particular also extending previous considerations to very late times. The implicit LES capability for moderate Reynolds number ranges is assessed by the Comte–Bellot Corrsin decaying grid generated isotropic turbulence. The evolution of shear layer instabilities into and throughout highly nonlinear stages can be studied by considering the two-dimensional doubly-periodic shear layer with finite thickness, further extended to infinitely thin shear-layers at infinite Reynolds numbers. The interaction between large scale vortical structures with the very-small-scale structures of viscous boundary layers walls is studied by considering an isolated vortex dipole colliding with a no-slip wall, following Refs. [13–16]. The lid-driven cavity is discussed as an example for a fully confined wall-bounded non-turbulent, but with respect to proper numerical resolution highly demanding two-dimensional flow.

2. Model formulation

2.1. Artificial compressibility approach

At Mach numbers $M \ll 1$ compressibility is negligible, i.e. $\beta = \frac{1}{\rho}\frac{\partial\rho}{\partial p} \approx 0$. The *artificial compressibility* approach of Chorin and Temam [17,18] assumes a nonzero but constant compressiblity for weakly compressible flows. The isentropic compressibility relates to the sound speed by $a^2 = \frac{1}{\rho\beta|_s}$. For flows with $M = 0.1$, as considered within this work, the isentropic compressibility is on the order of $\beta|_s = 0.01$. For isothermal processes $\beta = \beta|_s$, and the ratio of specific heats is $\gamma = 1$. Pressure and density are directly related by

$$p = a^2\rho. \tag{1}$$

It is evident that density fluctuations can be considered as small, if a is a sufficiently large constant.

2.2. Numerical-flux computation adapted to weakly compressible fluid treatment

Within the weakly compressible approach total energy is determined by the evolution of mechanical energy. Thus, the flow is governed by equations for the conservation of mass and momentum. In one dimension (for simplicity) $\mathbf{u} = (\rho, \rho u)$ is the solution of

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial}{\partial x}\mathbf{f}(\mathbf{u}) = 0. \tag{2}$$

In a discrete space–time-domain, the discrete conservation equation

$$\frac{d\mathbf{U}_i}{dt} = -\frac{1}{\Delta x_i}\left(\mathbf{F}\left(\mathbf{u}\left(x_{i+\frac{1}{2}}, t\right)\right) - \mathbf{F}\left(\mathbf{u}\left(x_{i-\frac{1}{2}}, t\right)\right)\right), \tag{3}$$

for the cell-averaged solution $\mathbf{U}_i$ requires approximations of the cell-face fluxes $\mathbf{F}_{i\pm\frac{1}{2}}$. A straightforward low-dissipation flux approximation is due to the Roe [19] approximate Riemann solver. Successful applications of Roe schemes to the solution of weakly compressible flows have been demonstrated by Marx [20] and Elsworth and Toro [21].

Roe's linearization of the local flux Jacobian $\tilde{\mathbf{A}}_j = \tilde{\mathbf{A}}(\hat{u}_L, \hat{u}_R)$ is essential. The eigenvalues of $\tilde{\mathbf{A}}_j$ are $\tilde{\lambda}_j(\hat{u}_L, \hat{u}_R)$ and its right eigenvectors $\tilde{\mathbf{K}}^{(j)}(\hat{u}_L, \hat{u}_R)$ are determined so that the Roe numerical flux function can be computed as

$$\hat{\mathbf{F}}_{i+\frac{1}{2}} = \frac{1}{2}(\hat{f}_L + \hat{f}_R) - \frac{1}{2}\sum_{j=1}^{m}\tilde{\alpha}_j|\tilde{\lambda}_j|\tilde{\mathbf{K}}^{(j)}. \tag{4}$$

Using the left and right reconstructed states $\hat{\mathbf{u}}_L$ and $\hat{\mathbf{u}}_R$ at the interface $i = \frac{1}{2}$, the procedure to compute the eigenvalues $\tilde{\lambda}_j$, right eigenvectors $\tilde{\mathbf{K}}^{(j)}$ and wave speeds $\tilde{\alpha}_j$ is straightforward. The Roe averaged density $\tilde{\rho}$ and velocity $\tilde{u}$ are obtained from the left and right states as

$$\begin{aligned} \tilde{\rho} &= \sqrt{\rho_L\rho_R}, \\ \tilde{u} &= \frac{\sqrt{\rho_L}u_L + \sqrt{\rho_R}u_R}{\sqrt{\rho_L} + \sqrt{\rho_R}}. \end{aligned} \tag{5}$$

Within the weakly compressible approach the local Roe-averaged speed of sound $\tilde{a}$ is replaced by the constant speed of sound a. Thus, $\tilde{\lambda}_j, \tilde{\mathbf{K}}^{(j)}$ and $\tilde{\alpha}_j$ are:

$$\tilde{\lambda}_1 = \tilde{u} - a, \quad \tilde{\lambda}_2 = \tilde{u} + a, \tag{6}$$

$$\tilde{\mathbf{K}}^{(1)} = \begin{bmatrix} 1 \\ \tilde{u} - a \end{bmatrix}, \quad \tilde{\mathbf{K}}^{(2)} = \begin{bmatrix} 1 \\ \tilde{u} + a \end{bmatrix}, \tag{7}$$

$$\begin{aligned} \tilde{\alpha}_1 &= \frac{1}{2a^2}[(p_R - p_L) - \tilde{\rho}a(\hat{u}_R - \hat{u}_L)], \\ \tilde{\alpha}_2 &= \frac{1}{2a^2}[(p_R - p_L) + \tilde{\rho}a(\hat{u}_R - \hat{u}_L)]. \end{aligned} \tag{8}$$

The resulting semi-discrete evolution equation

$$\frac{d\mathbf{U}_i}{dt} = -\frac{1}{\Delta x_i}\left(\hat{\mathbf{F}}\left(x_{i+\frac{1}{2}}, t\right) - \hat{\mathbf{F}}\left(x_{i-\frac{1}{2}}, t\right)\right), \tag{9}$$

can be advanced in time with a three-step TVD Runge–Kutta scheme [22].