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Mixed convection boundary-layer flow past a horizontal circular cylinder embedded in a porous medium filled with a nanofluid under convective boundary condition

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ABSTRACT

The steady mixed convection boundary-layer flow past a horizontal circular cylinder in a stream flowing vertically upwards embedded in porous medium filled with a nanofluid is studied, taking into account the thermal convective boundary condition is studied. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. The governing partial differential equations are transformed into a set of non-similar equations and solved numerically by an efficient implicit, iterative, finite-difference method. Comparisons with previously published work are performed and excellent agreement is obtained. A parametric study of the physical parameters is conducted and a representative set of numerical results for the velocity, temperature, and nanoparticle volume fraction profiles as well as the local skin-friction coefficient, local Nusselt and Sherwood numbers is illustrated graphically to show interesting features of the solutions.

1. Introduction

Mixed convection boundary-layer flow and heat transfer in fluid saturated porous media occurs in many applications of engineering. The interest in this subject has been stimulated, to a large extent, by the fact that thermally driven flows in porous media have considerable applications in mechanical, chemical and civil engineering. Applications include fibrous insulation, food processing and storage, thermal insulation of buildings, geophysical systems, electro chemistry, metallurgy, the design of pebble bed nuclear reactors, underground disposal of nuclear or non-nuclear waste, solar power collectors, geothermal applications, nuclear reactors, etc. Theories and experiments of thermal convection in porous media and state-of-the-art reviews with special emphasis on practical applications are presented in the recent books by, Pop and Ingham [1], Ingham and Pop [2], Vafai [3] and Nield and Bejan [4].

On other hand, the term of nanofluid refers to a solid–liquid mixture with a continuous phase which is a nanometer sized nanoparticle dispersed in conventional base fluids. In order to study the heat transfer behavior of the nanofluids, precise values of thermal and physical properties such as specific heat, viscosity and thermal conductivity of the nanofluids are required. There are a few well-known correlations for predicting the thermal and physical properties of nanofluids which are often cited by researchers to calculate the convective heat transfer behaviors of the nanofluids. Each researcher has used different models of the thermophysical properties in their works. Convective heat transfer in nanofluids is a topic of major contemporary interest both in sciences and engineering. Several ordinary fluids including water, toluene, ethylene glycol and mineral oils, etc. in heat transfer processes have rather low thermal conductivity. The nanofluid (initially introduced by Choi [5]) is an advance type of fluid containing nanometer sized particles (diameter less than 100 nm) or fibers suspended in the ordinary fluid. Undoubtedly, the nanofluids are advantageous in the sense that they are more stable and have acceptable viscosity and better wetting, spreading and dispersion properties on solid surface. Nanofluids are used in different engineering applications such as microelectronics, microfluidics, transportation, biomedical, solid-state lighting and manufacturing. Furthermore, suspensions of metal nanoparticles are also being developed for other purposes, such as medical applications including cancer therapy. The interdisciplinary nature of nanofluid research presents a great opportunity for exploration and discovery at the frontiers of nanotechnology. A comprehensive survey of convective transport in nanofluids was made by Buongiorno [6]. Abu-Nada et al. [7] reported on the heat transfer enhancement in combined convection around a rotating horizontal cylinder using nanofluids. Kuznetsov

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and Nield [8] have studied the classical problem of free convection boundary layer flow of a Newtonian fluid past a vertical flat plate in a porous medium saturated by a nanofluid. Abu-Nada [9] investigated the effects of variable viscosity and thermal conductivity of Al_2O_3–water nanofluid on heat transfer enhancement in horizontal annuli. Ahmed and Pop [10] studied the steady mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium filled with nanofluids. Chamkha et al. [11] have also analyzed the natural convection past a sphere embedded in a porous medium saturated by a nanofluid. Nazar et al. [12] studied steady mixed convection boundary layer flow from a circular cylinder embedded in a porous medium filled with a nanofluid. A similarity analysis for the problem of a steady boundary-layer flow of a nanofluid on an isothermal stretching circular cylindrical surface is presented by Gorla et al. [13]. Gorla et al. [14] have also studied mixed convective boundary layer flow over a vertical wedge embedded in a porous medium saturated with a nanofluid. Chamkha et al. [15] analyzed the effect of melting on unsteady hydromagnetic flow of a nanofluid past a stretching sheet. The problem of the natural convection boundary layer of a non-Newtonian fluid about a permeable vertical cone embedded in a porous medium saturated with nanofluid has been performed by Rashad et al. [16]. Chamkha et al. [17,18] investigated the effect of radiation on mixed convection over a wedge and cone embedded in a porous medium filled with a nanofluid. Chamkha and Rashad [19] studied the natural convection from a vertical permeable cone in nanofluid saturated porous media for uniform heat and nanoparticles volume fraction fluxes. Chamkha et al. [20] have studied the effect of radiation on boundary-layer flow of a nanofluid on a continuously moving or fixed permeable surface. Chamkha et al. [21] have analyzed the transient natural convection flow of a nanofluid over a vertical cylinder. Modather and Chamkha [22] investigated the effect of double diffusion on mixed convection in an axisymmetric stagnation flow of a nanofluid past a vertical cylinder.

The objective of the present study is to analyze the mixed convection boundary-layer flow past a horizontal circular cylinder in a stream flowing vertically upwards embedded in porous medium filled with a nanofluid taking into account the thermal convective boundary condition. The effects of Brownian motion and thermophoresis are included for the nanofluid. The coupled nonlinear partial differential equations are solved numerically using an implicit finite difference scheme. The effects of relevant parameters on the dimensionless nanofluid velocity, the temperature, the nanoparticle volume fraction, as well as the local skin-friction coefficient, local Nusselt number and local Sherwood number are investigated and shown graphically and discussed.

2. Mathematical formulation

Consider a steady mixed convection boundary layer flow past a horizontal circular cylinder of radius a in a stream flowing vertically upwards embedded in porous medium filled with a nanofluid taking into account the thermal convective boundary condition. It is assumed that the uniform temperature of the cylinder surface is T_f and that of the nanofluid volume fraction is ϕ_w, while the uniform temperature and nanofluid volume fraction in the ambient nanofluid are T_∞ and ϕ_∞, respectively. The cylinder is either heated or cooled by convection from a fluid of temperature T_f such that $T_f > T_\infty$ corresponding to a heated surface (assisting flow) and $T_f < T_\infty$ corresponding to a cooled surface (opposing flow) respectively, where the velocity of the external flow is $\bar{u}_e(x)$. Fig. 1 shows the flow model and physical coordinate system. The orthogonal coordinates x and y are measured along the surface of the cylinder, starting with the lower stagnation point, and normal to it, respectively. In order to study transport through non-Darcian porous medium, the Brinkman-extended Darcy model is used, where the velocity square term is neglected (see, Vafai and Tien [23]). It is assumed that the Boussinesq and boundary layer approximations are valid. Under these assumptions, the equations governing the steady mixed convection boundary-layer flow are; (see Merkin [24] and Nazar et al. [25])

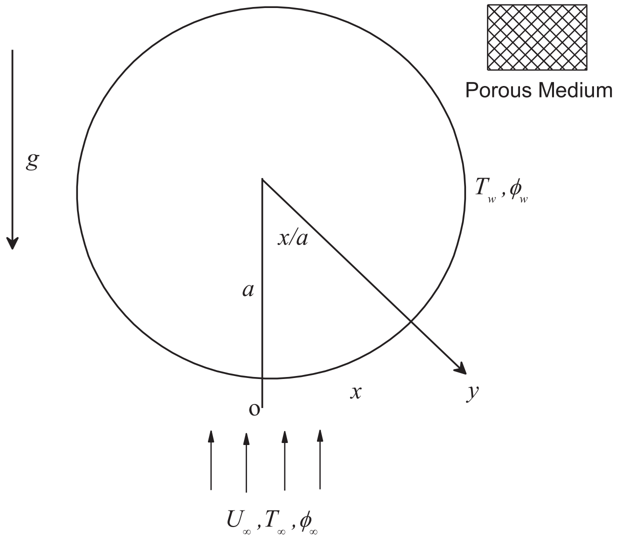

Fig. 1. Flow model and coordinate system.

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0, \tag{1}$$

$$\begin{aligned}\rho_{f\infty}\left(\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y}\right) &= \rho_{f\infty}\bar{u}_e\frac{\mathrm{d}\bar{u}_e}{\mathrm{d}x} + \mu\frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\mu}{K}(\bar{u}_e - \bar{u}) \\ &\quad + [(1-\phi_\infty)g\rho_{f\infty}\beta(T - T_\infty) \\ &\quad - (\rho_p - \rho_{f\infty})g(\phi - \phi_\infty)]\sin\left(\frac{x}{a}\right),\end{aligned} \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial x} + \bar{v}\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial \phi}{\partial y}\frac{\partial T}{\partial y} - \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{3}$$

$$\bar{u}\frac{\partial \phi}{\partial x} + \bar{v}\frac{\partial \phi}{\partial y} = D_B\frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

subject to the boundary conditions;

$$\begin{aligned}&\bar{u} = \bar{v} = 0, \quad -k\frac{\partial T}{\partial y} = h_f(T_f - T), \quad \phi = \phi_w \text{ on } y = 0 \\ &\bar{u} \to \bar{u}_e(x), \quad T \to T_\infty, \quad \phi \to \phi_\infty \text{ on } y \to \infty,\end{aligned} \tag{5}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components along x and y axes, respectively, T is the temperature in the fluid phase, ϕ is the nanoparticle volume fraction, K is the permeability of the porous medium, ρ, μ, k, g, and β are the density, viscosity, thermal conductivity, gravitational acceleration and volumetric thermal expansion coefficient of the nanofluid, $\rho_{f\infty}$ is the density of the base fluid, ρ_p is the density of the nanoparticles, $\alpha_m = k/(\rho C)_f$ is the thermal diffusivity of the fluid, $\tau = (\rho C)_p/(\rho C)_f$ is the nanofluid heat capacity ratio, $(\rho C)_f$ is the heat capacity of the fluid and $(\rho C)_p$ is the effective heat capacity of the nanoparticle material. D_B the Brownian diffusion coefficient, D_T the thermophoretic diffusion coefficient, h_f is the convective heat transfer coefficient and the subscripts w and ∞ indicate the conditions at the surface and at the outer edge of the boundary layer respectively. Following see Merkin [24] and Nazar et al. [25] that the velocity of the external flow $\bar{u}_e(x)$