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The effect of double stratification on boundary-layer flow and heat transfer of nanofluid over a vertical plate

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ABSTRACT

The problem of double stratification on boundary layer flow and heat transfer induced due to a nanofluid over a vertical plate is investigated. The transport equations employed in the analysis include the effect of Brownian motion, thermophoresis, thermal stratification and solutal stratification parameters. The nonlinear governing equations and their associated boundary conditions are initially cast into dimensionless forms by similarity variables. The resulting systems of equations are then solved numerically using Keller-box method. The solution for the temperature and nanoparticle concentration depends on parameters viz. thermal and solutal stratification parameters, Prandtl number *Pr*, Lewis number *Le*, Brownian motion *Nb*, buoyancy ratio parameter *Nr* and the thermophoresis parameter *Nt*. Numerical results are obtained for velocity, temperature and concentration distribution as well as the local Nusselt number and Sherwood number. It is found that the local Nusselt number and Sherwood number decrease with an increase in stratification parameters ε_1 and ε_2. However, the skin friction coefficient $f''(0)$ increases with an increase in mass stratification parameter ε_2 and decreases with an increase in thermal stratification parameter ε_1.

The obtained results are displayed both graphically tabular form to illustrate the effect of the stratification parameters on the dimensionless velocity, wall temperature and concentration. The numerical results are compared and found to be in good agreement with previous published result on special cases of the problem.

1. Introduction

Natural convection flows have received a great interest because of their various engineering and industrial applications in heat transfer process. In nature, convection cells formed from air arising above sunlight-warmed land or water are a major feature of all weather systems. Convection is also seen in the rising plume of hot air from fire, oceanic currents, and sea-wind formation (where upward convection is also modified by Coriolis forces). In engineering applications, convection is commonly visualized in the formation of micro-structures during the cooling of molten metals, and fluid flows around shrouded heat-dissipation fins, and solar ponds. A very common industrial application of natural convection is free air cooling without the aid of fans: this can happen on small scales (computer chips) to large scale process equipment.

Heat transfer by natural convection frequently occurs in many physical problems and engineering applications such as geothermal systems, heat exchangers, chemical catalytic reactors, fiber and granular insulation, packed beds, petroleum reservoirs and nuclear waste repositories. Moreover, natural convection of heat and mass transfer occurs in many areas. For instance, it occurs in the fields of design of chemical processing equipment, formation and dispersion of fog, distributions of temperature and moisture over agricultural fields and groves of fruit trees, and damage of crops due to freezing and pollution of environment. Natural convection from a vertical surface in a constant density medium has been widely studied for more than a century. Because of its wide application, still it is active research area in engineering disciplines.

The effect of stratification is an important aspect in heat and mass transfer, and it has been studied by several researchers. Stratification of fluids arises due to temperature variation, concentration differences, or presence of different fluids of different densities. In practical situations where heat and mass transfer run simultaneously, it is important to investigate the effect double stratification on the convective transport in nanofluids.

Stratified fluids are ubiquitous in nature, present in almost any heterogeneous fluid body. Examples include thermal stratification of reservoirs and oceans, salinity stratification in estuaries, rivers, groundwater reservoirs, and oceans, heterogeneous mixtures in industrial, food, and manufacturing processing, density

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Nomenclature

C_f	skin friction coefficient
C_∞; 0	ambient concentration
D_B	Brownian diffusion coefficient
D_T	thermophoresis diffusion coefficient
f	dimensionless stream function
h	dimensionless concentration function
k	thermal conductivity
Le	Lewis number
Nb	Brownian motion parameter
Nr	buoyancy ratio parameter
Nt	thermophoresis parameter
Nu_x	local Nusselt number
Pr	Prandtl number
Ra_x	local Rayleigh number
Re	local Reynolds number
Sh_x	local Sherwood number
T	temperature of the fluid inside the boundary layer
T_∞; 0	ambient temperature
u, v	velocity component along x- and y-direction component along x- and y-direction

Greek symbols

α	thermal diffusivity
β	volume expansion coefficient
η	dimensionless similarity variable
μ	dynamic viscosity of the fluid
υ	kinematic viscosity of the fluid
ε_1	thermal stratification
ε_2	solutal stratification
$(\rho)_f$	density of the fluid
$(\rho\ c)_f$	heat capacity of the fluid
$(\rho\ c)_p$	effective heat capacity of a nanoparticle
ψ	stream function
σ	electrical conductivity
ϕ_w	nanoparticle volume fraction at the surface
ϕ_∞	nanoparticle volume fraction at large values of y
θ	dimensionless temperature
τ	$\dfrac{(\rho c)_p}{(\rho c)_f}$

Subscripts

∞	condition at the free stream
W	condition at the surface

stratification of the atmosphere, and uncountable similar examples. In the presence of gravity, these density differences have a dramatic impact on the dynamics and mixing of heterogeneous fluids. For example, thermal stratification in reservoirs can reduce the vertical mixing of oxygen to the point that bottom water becomes anoxic through the action of biological processes. Preventing, predicting, and solving such a reservoir problem, though dependent on other limnological factors, requires an understanding of the dynamics of stratified fluids.

The notion of stratification is essential in lakes and ponds. It is important to control the temperature stratification and concentration differences of hydrogen and oxygen in such environments as they may directly affect the growth rate of all cultured species. Also, the analysis of thermal stratification is important for solar engineering because higher energy efficiency can be achieved with better stratification. It has been shown by researchers that thermal stratification in energy storage may considerably increase system performance. Accordingly, Chen and Eichhorn [1] analyzed natural convective flow over a heated vertical surface in a thermal stratified medium using the local non-similarity method for the solution of the governing equations. Kulkarni et al. [2] have obtained a similarity solution for the natural convection flow over a heated vertical plate in a thermally stratified fluid. Moreover, Angirasa and Srinivasan [3] have presented a numerical study of the natural convection flow on a vertical surface due to the combined effect of buoyancy forces caused by the heat and mass diffusion in a thermally stratified medium. Furthermore, the natural convection flow over a horizontal cylinder and a sphere immersed in a thermally stratified fluid was investigated by Chen and Eichhorn [4]. Similarly, Ishak et al. [5] studied mixed convection boundary layer flow adjacent to a vertical surface embedded in a stable stratified medium using implicit finite difference method. Also, Mukhopadhyay et al. [6] analyzed the effects of thermal stratification on flow and heat transfer past a porous vertical stretching surface using Lie group transformation method.

All the above studies analyze the effects in thermally stratified medium for common fluids; very few studies have been done to investigate the influence of thermal and mass stratification on heat and mass transfer by natural convective flow for non-Newtonian fluid. In view of these, some authors have studied doubly stratified effect in natural convective flow. Accordingly, Srinivasacharya and RamReddy [7–9] investigated numerically the effect of double stratification on free convection in a micropolar fluid for non-porous and non-Darcian porous plate.

All the aforementioned studies analyzed natural convective flow of common fluids. However, nowadays, the natural convective flow and heat transfer using a suspension of nanometer sized solid particles in base fluid has been active research area for more than two decades. Studies have shown that a nano particle suspension in a base fluid remarkably changes the transport property and heat transfer characteristics of a convectional base fluid. A uniform suspension of a nanometer-size solid particle and fibers in a convectional base fluid is called nanofluid [10]. This new kind of engineered colloidal suspension of nanoparticles in base fluid is mainly designed for a purpose of heat transfer enhancement. Accordingly, Khan and Aziz [11] numerically investigated natural convection flow of a nanofluid over a vertical plate with uniform surface heat flux using a fourth order Runge–Kutta method.

Nanofluid has been studied both experimentally and theoretically to get improved heat transport properties and higher energy efficiency in a variety of thermal exchange system for different industrial applications. This new kind of fluid has attracted the attention of many researchers because of its thermal enhancement property.

The studies on boundary layer flow and convective heat transfer in a nanofluid is a topic of contemporary research area in fluid science and engineering. Owing to its novelty, many research papers have been published on this area. Accordingly, Kuznetsov and Nield [12] have studied the natural convective boundary-layer flow of a nanofluid past a vertical plate analytically. They used the model in which Brownian motion and thermophoresis effects were taken into account. Their result indicates that the local Nusselt number is a decreasing function of the parameters such as buoyancy ratio, Brownian motion and thermophoresis parameter.