Computers & Fluids 86 (2013) 510–536

Contents lists available at ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

SPH for incompressible free-surface flows. Part II: Performance of a modified SPH method

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ARTICLE INFO

Article history:
Received 8 January 2013
Accepted 15 July 2013
Available online 24 July 2013

Keywords:
SPH
Free-surface flows
Weak compressibility assumption
Kernel interpolation
Instability
High-frequency oscillations
Treatments
Smoothing
Liquid column collapse
Dam-break
Sloshing

ABSTRACT

Building on Part I, we propose a modified method, mSPH, which retains the weak compressibility and kernel interpolation of the basic SPH method, but suppresses the two sources of spurious high-frequency dynamics in the presence of weak compressibility: the (stable) acoustic eigen solutions, and the unstable depth-oscillatory modes (associated with non-uniform density). We achieve this by using a form of the initial and boundary conditions consistent with the governing equations; and employing a robust dissipative scheme, in the form of periodic smoothing. We quantify the effect and efficacy of the latter in terms of the numerical parameters: the artificial sound speed, the kernel bandwidth, the Courant condition, and the smoothing frequency. Further, we obtain a global error metric that quantifies the spectral amplitudes of the high-frequency dynamics and identifies the initiation and growth of temporally unstable modes. This metric is used as an independent measure for the validity of the weak compressibility assumption, without the need for calibration with external data. We demonstrate the performance of mSPH, and the usefulness of the error metric in four illustrative applications: the hydrostatic problem, the collapse of a liquid column, the standard dam-break benchmark, and sloshing in a swaying tank. It is shown that mSPH is robust and obtains convergent and accurate kinematics and dynamics compared to theory and experiments.

1. Introduction

Smoothed Particle Hydrodynamics (SPH) has been widely used for incompressible, free-surface hydrodynamic problems [31] and especially on violent flows (see e.g., [50,48,5,32,34,50,46,55,23,8]) where there are few effective alternatives. Because of the weak compressibility assumption and the spatial discretization technique kernel interpolation (KI), which are inherent in the basic SPH formulation, simulation results can be affected by large high-frequency oscillations (HFO) in the dynamics [12,44,11,7,19,41,46,39].

Because of these inherent issues in the method, SPH is rarely used in its basic form without treatments. To address the HFO, reformulations and treatments to the weak compressibility, KI, and associated instability mechanisms have been developed with varying degrees of success. Treatments to KI include reformulations of the governing equations [27,22,42,25,21], and modifications to or replacement of KI with alternative schemes [3,18,47,53,26]. The former aim to improve conservation properties of the scheme, but generally do not eliminate the generation of HFO near the free surface. The latter could reduce the initial generation of HFO near the free surface but may become locally singular requiring further treatments (see e.g., [3,18]). Even though KI is the root cause of the initial generation of HFO near the free surface (Section 4, Part I) it remains second-order consistent even for non-uniform distributions, and is not the cause or sole source of HFO in the presence of weak compressibility (Sections 3 and 5, Part I).

To address the inherent instabilities, whether attributed primarily to KI or weak incompressibility (e.g., [14,33,16,37,36,54,15]; [49]; Part I Section 5), common strategies include periodic density re-initialization schemes via smoothing [1,47,50,5], and the addition of small, tunable term in a governing equation. The former schemes are robust without involving computation of spatial derivatives, but the effects of the re-initialization can be sensitive to the order of the smoothing [6]. Examples of the latter approach include: artificial viscosity [31,5] and tensile instability treatment [33] operating on the momentum equation; XSPH velocity [30] and initial damping [31] operating on the particle positions; diffusive terms [12,44,45,3] operating on the mass conservation; and inclusion of energy equation [12,13]. Addition of a small term in a governing equation may result in problem-specific efficacy. For example, the tensile instability treatment [33] is successful in magnetohydrodynamic applications but has shown little stability improvements in free-surface applications [5,33] where the hydrostatic density distribution significantly alters the instability mechanisms. Recent preferred approaches involve addition of dissipative terms in both

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http://dx.doi.org/10.1016/j.compfluid.2013.07.016

the momentum and mass conservation equations [12,44,45,3]. These obtain some increase in performance, but may also have problem-specific efficacy.

Another category of treatments involve some form of low-pass filtering of the HFO by post-processing the dynamics in either the spatial [19,41,11,46,39] or temporal domains [8,7]. While the former are not robust [39], the latter are effective in removing the HFO without however, eliminating the effect of the instability on the obtained results.

Current state-of-the-art SPH codes (such as SPHysics) include a collection of such treatments, with options for including or excluding the treatments, to address problem-specific issues. In this context, the precise mechanisms involved and the effects of the different treatments on the simulated physics are often not well understood or characterized, while the choice of the tunable parameters may necessitate calibrations against external data.

We note also the development of variations of the SPH approach which remove the fundamental assumption of weak compressibility from SPH. Notable examples include the moving-particle semi-implicit method (MPS) [51,23,24,40,38,39], where a Poisson equation for pressure is solved to ensure incompressibility in terms of particle densities; and the SPH projection method [52]. These are beyond the immediate interest of the present work.

The objective of this paper is to develop rational modifications of the SPH algorithm based on the analyses and findings in Part I. Without revisiting the underlying assumptions of weak compressibility and kernel interpolation (KI) of the original SPH method, we focus on targeted treatments of the basic algorithm that effectively suppress the HFO dynamics. Of particular interest is understanding of the mechanisms introduced by the treatments in terms of the stability and consistency issues identified in Part I. For useful applications, it is important also to obtain precise quantifications of the effects of the treatment on the simulated physics, and ideally to obtain accuracy metrics that allow the HFO dynamics to be minimized without the need for comparison to external reference data.

Building on the analyses and results of Part I, we develop a modified SPH (mSPH) scheme that obtains robust, accurate and stable kinematics and dynamics for short- and long-time simulations of simple and complex free-surface flows. The key modifications to the basic SPH algorithm are enforcements of the initial and boundary conditions that are consistent with the governing equations, and periodic smoothing of the velocity and density fields.

Even though KI is the root cause of the initial generation of HFO near the free surface, we find that initial and solid boundary conditions that are inconsistent to the governing equations generate HFO of comparable amplitudes with those at the free surface. Moreover, HFO generated at solid boundaries are continuously introduced throughout the duration of the simulation. Thus, enforcing consistent initial (e.g., [20]) and boundary conditions results to an appreciable reduction in the generation of HFO. Low-order periodic smoothing [50,47] is shown to introduce a small numerical dissipation to the density in a robust manner. As in [12,44,45,3], the efficacy is increased when the dissipation is applied to both the velocity and density fields. Based on the stability analysis framework of Part I, we quantify and control the numerical dissipation due to smoothing in terms of the numerical parameters. Together these simple modifications suppress the generation and unstable growth of HFO everywhere in the SPH domain, with a small, robust, and quantified and controlled effect on the simulated physics.

Based on the analyses in Part I which elucidate the frequencies and growth rates of the HFO, we are able to define a global error metric which quantifies the fidelity of the simulation. This metric is a single, independent quantity from the simulations that can be used for controlling and optimizing the numerical parameters of mSPH without the need for calibration using known solutions.

The robustness and accuracy of mSPH as well as the use of the global error metric are illustrated in a number of benchmark problems: solution of the hydrostatic case (compared to that using the basic SPH method); a collapsing liquid column (quantifying the effect of dissipation on the kinematics); the dam-break problem (obtaining accurate kinematics as well as dynamics, illustrating the efficacy of using the global error metric); sloshing in a swaying tank (demonstrating long-time high fidelity stable and accurate dynamics). In all these cases, mSPH obtains stable and smooth solutions for both the kinematics and dynamics which are in good agreement with known results and/or experiments. In every case, HFO are effectively eliminated based on the use of the single error metric we define, which quantities the fidelity of the simulation without reference to independent external data.

The structure of Part II is as follows. In Section 2 we formulate the modified SPH scheme (mSPH) and introduce the global error metric. Section 3 contains benchmark examples of the performance of mSPH simulations. Conclusions are given in Section 4.

2. Rational modifications: developing the modified SPH (mSPH) scheme

We propose a modified SPH (mSPH) scheme that retains the simplicity allowed by the weak compressibility assumption and meshless nature of KI. The main objective is to minimize the presence of the HFO in the simulation. We modify the typically used initial and boundary conditions to reduce associated acoustic solutions (Part I, Section 3). We smooth the velocities and densities to dissipate all manifestations of HFO and we obtain a quantification of the effective dissipation due to the smoothing. We introduce a global error metric that assesses the amplitude and stability of the HFO and determines optimal numerical parameters that sufficiently suppress unstable HFO without over-dissipating desirable solutions. The mSPH scheme significantly eliminates HFO to obtain accurate kinematics and dynamical results that do not require calibration of the numerical parameters with external solutions.

2.1. Algorithmic modifications

2.1.1. Governing equations

In cartesian coordinates $\mathbf{x}$ the momentum for an incompressible inviscid fluid with density ρ_f, gravity $\mathbf{g}$ and velocity $\mathbf{u}$ is described by the Euler equation:

$$\frac{d\mathbf{u}}{dt} = -\frac{\nabla P}{\rho_f} - \mathbf{g}. \tag{1}$$

The weak compressibility assumption obtains the pressure P through an equation of state (EoS), a numerical 'particle' density ρ, and an (artificial) sound speed c. The linearized EoS is:

$$dP = c^2 d\rho, \tag{2}$$

and ρ satisfies mass conservation:

$$\frac{d\rho}{dt} = -\rho\nabla\cdot\mathbf{u}. \tag{3}$$

Note that here the role of ρ is only numerical, serving as an intermediate step for the computation of the pressure, similarly to a single-iteration pseudo-compressibility method [35,10]. The momentum in (1) is $\rho_f d\mathbf{u}/dt$. For comparison it is noted that, the momentum in the basic SPH is $\rho d\mathbf{u}/dt$. This difference has only higher-order effects with respect to density fluctuations $\Delta r \equiv (\rho - \rho_f)/\rho_f = O(c^{-2})$. Substituting Δr into the pressure term in (1) and expanding about $\Delta r = 0$ obtains $\nabla P/\rho_f = \nabla\ P/\rho + O(c^{-2})$. Therefore, the weakly compressible analysis and corresponding