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Recursive nullspace-based control allocation with strict prioritization for marine craft ⋆

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Abstract: In control allocation it is often important to prioritize among the directions to produce control effort, especially in cases of limited capacity. Motivated by a requirement for strict prioritization among the control directions, a recursive nullspace-based allocation design based on direct use of the generalized pseudoinverses and nullspace matrices is proposed. The allocation design divides the overall problem into r subproblems according to a prioritization sequence, and a recursive method is proposed to solve the allocation subproblem in each step. By hiding the allocated controls from subsequent steps in the nullspace corresponding to the present step, the influence of lower priority control actions onto higher priority directions are nullified to achieve the specified prioritization. The method is verified by analyzing the thruster capacity of an Arctic intervention vessel based on experimental ice towing data.

Keywords: Control allocation; thrust allocation; motion control; dynamic positioning; Arctic operations.

1. INTRODUCTION

For dynamic positioning (DP) of offshore vessels, the limiting capacity of stationkeeping in heavy environmental conditions is the thruster configuration and the maximum resultant forces and moment that can be produced for surge, sway, and yaw motions (Sørensen, 2005). The DP control law calculates a commanded net force/moment to be produced by the propulsion system to compensate the environmental loads. A thrust allocation algorithms distributes this net force/moment in an efficient manner to a commanded force and direction for each individual thruster based on their rated power, locations in the hull, thruster types, and constraints. The yaw moment is typically prioritized before surge and sway for operational safety.

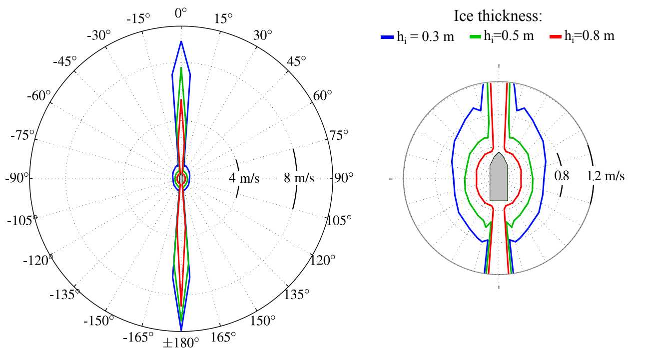

Fig. 1. DP-Ice Capability Plot of the CIVArctic vessel (left) with magnification (right), parametrized by ice drift speed entering at the respective angles. The 3 plots indicate 3 different ice thicknesses. (Courtesy: Su et al., 2013)

In Arctic DP operations of offshore vessels in ice (NTNU, 2010-2014), the ice loads on the vessel are generally larger and more aggressive than open water environmental forces. Moreover, vessels that operate in ice have typically been designed with an icebreaking bow or stern to minimize the loads and make stationkeeping feasible. This is clearly seen from the *DP-Ice Capability Plot* in Figure 1, presented by Su et al. (2013) for the customized design of an Arctic intervention vessel to operate partly in ice (Berg et al., 2011). This shows that if the ice enters outside the narrow longitudinal sectors from ahead or astern, then the icebreaking ability – and thus the stationkeeping capability – is significantly deteriorated. Consequently, prioritizing the yaw moment to keep the vessel heading (bow or stern) pointed against the ice drift becomes an even stricter requirement for DP in ice compared to open water (Moran et al., 2006). A good heading control will as a consequence minimize the sway loads, which therefore implies that the vessel must secondly prioritize surge forces – to push the vessel towards the incoming ice – before lastly the sway forces are given priority.

In general, having a commanded vector $\tau \in \mathbb{R}^m$ of net or resultant control effort related to the individual control effectors $u \in \mathbb{R}^n$ through the state (x)- and time (t)-dependent map $u \mapsto h(u, x, t) = \tau$, then *control allocation*, being the generalized notion of *thrust allocation* for marine vessels, is to calculate the individual distribution of control efforts in u. Having typically an overactuated setup, with $n > m$, there will be infinitely many solutions, and the objective is to find the "most optimal" $u \in \mathbb{U}$ where $\mathbb{U}$ is a constraint set for the control effectors.

Control allocation is a mature research field in aerospace applications (Oppenheimer et al., 2010) and for marine applications (Sørensen, 2005; Fossen, 2011), and is emerging also in other fields such as in process control and electrical power production systems. Solutions range from simple unconstrained pseudoinverse methods (Horn and Johnson, 1985; Fossen, 2011; Virnig and Bodden, 1994) to advanced constrained optimization-based methods such as linear programming, quadratic programming, nonlinear programming, and mixed-integer programming. The survey article by Johansen and Fossen (2013) gives an overview of such methods and their references.

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In this paper we consider the problem of prioritizing the resultant control directions in τ and propose a generic recursive algorithm for control allocation based on the generalized pseudoinverse method. We will first set up the control problem according to the desired sequence of priorities. Then a recursive control allocation design with r steps is conducted, where in each step a minimum norm solution (see Appendix for details) to an unconstrained control allocation subproblem is calculated, and a direct modification of the unconstrained solution is applied to mitigate the constraints set $\mathbb{U}$. The recursive design will ensure that the available control capacity is first used on the highest priority control direction, then the remaining capacity is used on the secondary direction, and so on. In other words, we avoid in cases where feasibility is not achievable to violate all directions in τ – but rather only those of lowest priority.

2. PROBLEM FORMULATION

Consider a control allocation problem based on a linear effector model,

$$\tau = Bu, \tag{1}$$

where $\tau \in \mathbb{R}^m$ is the resultant control input to the dynamic system, $u \in \mathbb{U} \subseteq \mathbb{R}^n$ is the vector of individual constrained control effectors to be allocated, $B \in \mathbb{R}^{m\times n}$ is the effector configuration matrix, and the constraint set $\mathbb{U}$ is given by p linear inequality constraints

$$\mathbb{U} = \left\{ u \in \mathbb{R}^n : \; Au \leq c, \; A \in \mathbb{R}^{p\times n}, \; c \in \mathbb{R}^p \right\}, \tag{2}$$

where

$$Au \leq c \;\Rightarrow\; \begin{bmatrix} a_1^\top \\ \vdots \\ a_p^\top \end{bmatrix} u \leq \begin{bmatrix} c_1 \\ \vdots \\ c_p \end{bmatrix}, \tag{3}$$

and $a_i^\top$ is the i'th row of A. With limited control capacity in u, the objective of the control allocation is to enforce a strict prioritization, based on an operational or safety consideration for the system under control, by first using the limited control capacity to satisfy the most important control direction(s) $\tau_1 \in \mathbb{R}^{m_1}$ in τ, then the second most important $\tau_2 \in \mathbb{R}^{m_2}$, and so on with $\tau_r \in \mathbb{R}^{m_r}$ having the lowest priority. Correspondingly, we ensure that the elements of τ and rows of B in (1) are ordered according to

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_r \end{bmatrix} = \begin{bmatrix} B_1^\top \\ B_2^\top \\ \vdots \\ B_r^\top \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}, \tag{4}$$

where $\tau_i \in \mathbb{R}^{m_i}$, $i = 1, \ldots, r$, is the prioritized sequence of subvectors in τ, $B_i \in \mathbb{R}^{n\times m_i}$, and $m_1 + m_2 + \ldots + m_r = m$. The objective is then to allocate the net control τ to the individual effectors in u where the control directions $\tau_1, \ldots, \tau_r$ should be satisfied in the respective prioritized sequence.

2.1 Motivating example: Yaw priority for CSE1

The model ship C/S Enterprise I (CSE1) is used for model-scale experiments in the NTNU model basin Marine Cybernetics Laboratory (MC Lab) (Skåtun, 2011). The thruster configuration, shown in Figure 2, consists of two aft Voith Schneider thrusters having circular thrust regions, and a bow tunnel thruster producing thrust in the transversal direction. Suppose the objective is to design a strict yaw-prioritized thrust allocation for CSE1. The linear effector model, sequenced according to yaw priority, is

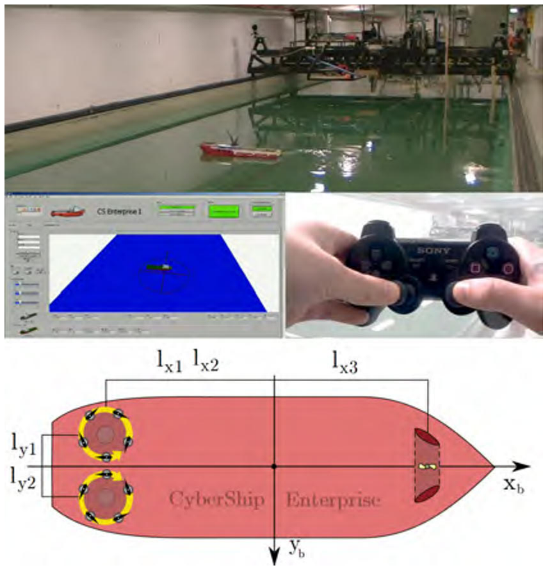

Fig. 2. Control system and thruster configuration for C/S Enterprise I (Courtesy: Skåtun, 2011).

$$\tau = Tf, \quad f = Ku, \quad \Rightarrow \quad \tau = TKu = Bu \tag{5}$$

$$\begin{bmatrix} \tau_N \\ \tau_X \\ \tau_Y \end{bmatrix} = \begin{bmatrix} -l_{1y} & l_{1x} & -l_{2y} & l_{2x} & l_{3x} \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \\ f_{3y} \end{bmatrix}$$

$$\begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \\ f_{3y} \end{bmatrix} = \begin{bmatrix} f_{1M} & 0 & 0 & 0 & 0 \\ 0 & f_{1M} & 0 & 0 & 0 \\ 0 & 0 & f_{2M} & 0 & 0 \\ 0 & 0 & 0 & f_{2M} & 0 \\ 0 & 0 & 0 & 0 & f_{3M} \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3y} \end{bmatrix}$$

where f_i are the individual thruster forces within maximum values f_{1M}, f_{2M}, and f_{3M}. The controls are scaled through the gains in K such that $|u_i| = 1 \Rightarrow |f_i| = f_{iM}$. To model the linear constriants $Au \leq c$ for CSE1, we divide the circular thrust regions into $2N$ linear curves $r_j u_x + u_y = \varsigma_j$, $j = 1, \ldots, 2N$ (where N is chosen freely), according to

$$r_j = \cot \frac{\pi}{2N}(2j+1)$$

$$\varsigma_j = u_{i,\max} \left[\sin \frac{\pi}{N}(j-1) + r_j \cos \frac{\pi}{N}(j-1) \right].$$

This gives $4N$ linear constraints for the Voith Schneider thrusters, that is, for $i = 1, 2$ we use

$$r_j u_{ix} + u_{iy} \leq \varsigma_j, \qquad j = 1, \ldots, N \tag{6a}$$

$$-r_j u_{ix} - u_{iy} \leq -\varsigma_j, \qquad j = N+1, \ldots, 2N. \tag{6b}$$

In addition, the tunnel thruster introduces the two constraints

$$u_{3y} \leq u_{3,\max} \tag{7a}$$

$$-u_{3y} \leq u_{3,\max}. \tag{7b}$$

Define $\tau_\psi := \tau_N$, $\tau_{xy} := \text{col}(\tau_X, \tau_Y)$ and, accordingly, $b_\psi^\top$ and $B_{xy}^\top$ such that

$$\begin{bmatrix} \tau_\psi \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} b_\psi^\top \\ B_{xy}^\top \end{bmatrix} u. \tag{8}$$

Let $u = v_{1a} + v_{1n}$ and target first $b_\psi^\top(v_{1a} + v_{1n}) = \tau_\psi$ in (8). Define the pseudoinverse $\left(b_\psi^\top\right)^\dagger := b_\psi \left[b_\psi^\top b_\psi\right]^{-1} \in \mathbb{R}^5$ and the corresponding nullspace orthogonal projection matrix $Q := I - \left(b_\psi^\top\right)^\dagger b_\psi^\top \in \mathbb{R}^{5\times 5}$ (see Appendix for details). Using