Computers & Fluids 86 (2013) 558–568

Contents lists available at ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

A coupled lattice Boltzmann-finite element approach for two-dimensional fluid–structure interaction

Alessandro De Rosis [a,*], Giacomo Falcucci [b], Stefano Ubertini [c], Francesco Ubertini [a]

[a] Department of Civil, Environmental and Materials Engineering (DICAM), University of Bologna, 40136 Bologna, Italy
[b] Department of Technologies, University of Naples "Parthenope", 80143 Naples, Italy
[c] Industrial Engineering School (DEIM), University of Tuscia, Largo dell'Universitá s.n.c., 01100 Viterbo, Italy

ARTICLE INFO

Article history:
Received 30 January 2013
Received in revised form 13 June 2013
Accepted 12 August 2013
Available online 22 August 2013

Keywords:
Fluid–structure interaction
Lattice Boltzmann method
Time Discontinuous Galerkin

ABSTRACT

In this paper, a systematic approach to couple the lattice Boltzmann and the finite element methods is presented for fluid–structure interaction problems. In particular, elastic structures and weakly compressible viscous fluids are considered. Three partitioned coupling strategies are proposed and the accuracy and convergence properties of the resultant algorithms are numerically investigated together with their computational efficiency. The corotational formulation is adopted to account for structure large displacements. The Time Discontinuous Galerkin method is used as time integration scheme for structure dynamics. The advantages over standard Newmark time integration schemes are discussed. In the lattice Boltzmann solver, an accurate curved boundary condition is implemented in order to properly define the structure position. In addition, moving boundaries are treated by an effective refill procedure.

1. Introduction

A reliable prediction of the interaction between fluids and structures is extremely important in several industrial, technological, biological and environmental processes. This great interest promoted a huge research effort in the last two decades. As well known, two different approaches, typical of any coupled problem, can be used to tackle fluid–structure interaction (FSI): monolithic approach and partitioned approach. Here, the partitioned approach is adopted, since it is the most suited for practical problems, [1]. The basic strategy is to treat separately the fluid and the structural domains and to properly discretize each of them, in order to adopt numerical methods developed and optimized for both computational fluid dynamics and computational structural dynamics. To meet the continuity conditions on the common fluid–structure boundary, an effective procedure should be devised to couple the two solvers.

In this work, weakly compressible viscous fluids are considered and the fluid flow is modeled via the Lattice Boltzmann method (LBM) [2]. In recent years, the LBM has proven to be a valid alternative to classical computational fluid dynamics based on the Navier–Stokes equations, as it has proved to be extremely simple and efficient, [2]. It is based on Boltzmann's kinetic equation, with its characteristic mesoscopic point of view, and not on Navier–Stokes *continuum* assumption, and is basically a fixed grid solve. The interested readers can refer to [3–7].

Complex boundary geometries were successfully simulated by means of the interpolation scheme described in [8,9] and recently LBM has been also used to solve problems with moving boundaries. In particular, Falucci et al. [10] investigated the flow physics induced by a rigid lamina undergoing moderately large harmonic oscillations. More recently the authors investigated the interaction of fluid flow with flexibly supported rigid bodies [11]. As far as structure dynamics, the Finite Element method (FEM) is adopted. Specifically, linear elastic slender structures idealized by beam finite elements are considered. Large displacements are accounted for by the corotational formulation [12] and time integration is performed by using higher order methods [13]. Among them, the Time Discontinuous Galerkin (TDG) method, implemented according to the two-stage algorithm recently proposed in [14], is selected. This choice is motivated by its good accuracy and stability properties, that are expected to play an important role in the performance of the whole procedure.

Although a huge effort has been devoted to fluid–structure interaction, only a few attention has been paid to coupling LBM for fluid dynamics and FEM for structure dynamics. LBM and FEM were successfully coupled for 2D fluid–structure interaction in [15]. Unlike the present work, plane finite elements were used to model the structure, together with the Newmark scheme for time integration. An interface mesh was introduced to adapt the two different discretizations and a staggered algorithm with subiterations was employed for the fluid. More recently, Lee et al.

* Corresponding author.
E-mail addresses: alessandro.derosis@unibo.it (A. De Rosis), giacomo.falcucci@uniparthenope.it (G. Falcucci), stefano.ubertini@unitus.it (S. Ubertini), francesco.ubertini@unibo.it (F. Ubertini).

0045-7930/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compfluid.2013.08.004

[16] proposed a coupled approach to investigate the propulsion effects of a flapping flexible plate in a quiescent fluid.

The main contribution of this paper is that it offers a systematic approach to couple LBM and FEM for fluid–structure interaction with slender structures. Three different coupling strategies for FSI are considered and their properties in terms of both accuracy and stability are numerically investigated. The same time discretization is adopted for both fluid and structure. The fluid is solved on a fixed grid (lattice) and the structure can move/deform upon the grid, similarly to immersed boundaries [17], with no restriction on matching between fluid and structure nodes (non-boundary-fitted method). This is one of the main advantages of this approach, since no moving meshes are needed that usually require high computational times. An accurate procedure to assign the curved boundary conditions [18] is adopted to account for the correct position of the structure. Moreover, upon structure motion, a simple refill procedure is used to initialize new activated fluid nodes. To compute the forces acting on the structure, the stress tensor at lattice sites is computed by the simple and effective procedure discussed in [19].

The first coupling algorithm proposed in this paper, called FELBA explicit, is very simple and can be classified as standard staggered algorithm. The basic idea has been recently applied by the authors in [11] to fluid–structure interaction with moving rigid bodies and is here generalized to couple LBM and FEM. The second coupling algorithm, called FELBA, is similar to the previous one apart from the introduction of a structure predictor, based on the idea proposed in [20,21] within a different framework, and from a different way to transfer fluid forces to the structure. The third one, called FELBA implicit, is obtained by the previous algorithm by iterating within each time step until a convergence criterion on interface conditions is met. It can be classified as strongly-coupled partitioned algorithm. In the presence of very light structure, Aitken's under-relaxation is used to prevent from potential instability due to added mass effects [22–24]. Numerical tests prove the reliability of the proposed approach and the good performance of all the proposed coupling algorithms. An almost optimal convergence rate is exhibited by the implicit version of FELBA, that is the convergence rate of the fluid flow with rigid structure. As expected, the structure predictor is proved to be very effective. The explicit algorithm with structure prediction is shown to exhibit a very good accuracy together with a nearly optimal computational cost involved, that is the cost involved by the sole fluid solver. Finally, the higher accuracy and stability provided by the TDG integration scheme are shown by means of comparisons with classical time integration schemes.

The paper is organized as follows. In Section 2, the problem is stated. In Section 3, the LB method is briefly presented. In Section 4, the TDG method and the corotational beam finite element are recalled. In Section 5, the coupling algorithms are discussed. Section 6 presents some numerical results on two different benchmark problems. Finally, Section 7 provides some concluding remarks.

2. Problem statement

A two-dimensional fluid–structure interaction problem is considered. The fluid is assumed to be viscous and weakly compressible. Fluid dynamics is modeled by a mesoscopic description using Boltzmann's kinetic equation [2]. Some details are given in Section 3.

The structure is assumed to be linearly elastic and geometrical non-linear. Structure dynamics is modeled using the classical shear undeformable beam theory. Some details are given in Section 4.

At the fluid–structure interface Γ, equilibrium and geometrical compatibility should be satisfied. Specifically, at each point of the interface the fluid velocity, $\boldsymbol{v}^{(f)}$, should be equal to the structure velocity, $\boldsymbol{v}^{(s)}$:

$$\boldsymbol{v}^{(f)} - \boldsymbol{v}^{(s)} = \mathbf{0} \quad \text{on } \Gamma. \tag{1}$$

In addition, the sum of tractions computed on the interface considered as fluid boundary, $\boldsymbol{t}^{(f)}$, and as structure boundary, $\boldsymbol{t}^{(s)}$, should be zero:

$$\boldsymbol{t}^{(f)} + \boldsymbol{t}^{(s)} = \mathbf{0} \quad \text{on } \Gamma. \tag{2}$$

3. Fluid modeling

In this section, the main aspects of the single-component, single-phase Lattice Boltzmann method for fluid dynamics modeling are presented. It is worth to notice that the Lattice Boltzmann method gives a mesoscopic description of fluid dynamics. The classical macroscopic continuum quantities such as fluid velocity and density are determined by stochastic-based upscaling procedure.

3.1. The Lattice Boltzmann method

Boltzmann's kinetic equation reads as follows:

$$\dot{f}(\boldsymbol{x}, \mathbf{v}, t) + \mathbf{v}(\boldsymbol{x}, t) \cdot \nabla f(\boldsymbol{x}, \mathbf{v}, t) = Q(\boldsymbol{x}, \mathbf{v}, t), \tag{3}$$

where $f = f(\boldsymbol{x}, \mathbf{v}, t)$ is the particle distribution function, that depends on space $\boldsymbol{x}$, particle velocity $\mathbf{v}$ and time t; the superimposed dot indicates differentiation with respect to time. Notice that particle velocity $\mathbf{v}$ differs from the macroscopic fluid velocity $\boldsymbol{v}$. $Q(\boldsymbol{x}, \mathbf{v}, t)$ is the collision operator that, based on the BGK simplified model, [25], can be defined as a relaxation towards a local equilibrium f^{eq}:

$$Q = \frac{1}{\tau}(f^{eq} - f), \tag{4}$$

with τ the relaxation parameter, which is strictly related to fluid viscosity.

Assuming a regular grid in space and time, as shown in Fig. 1, the discretized lattice Boltzmann equation is [2]:

$$f_j(\boldsymbol{x} + \Delta t \mathbf{c}_j, t + \Delta t) = f_j(\boldsymbol{x}, t) + \frac{1}{\tau}\left[f_j^{eq}(\boldsymbol{x}, t) - f_j(\boldsymbol{x}, t)\right], \tag{5}$$

where $\mathbf{c}_j$ is the jth particle velocity direction, $f_j = f_j(\boldsymbol{x}, t)$ and Δt is the time step. In this work, two-dimensional lattice grids and a standard 9-speed scheme (see Fig. 1) are adopted, [6]. It is worth to notice that Eq. (5) is explicit once f_j at the end of the current time step is expressed in terms of quantities that are known form the

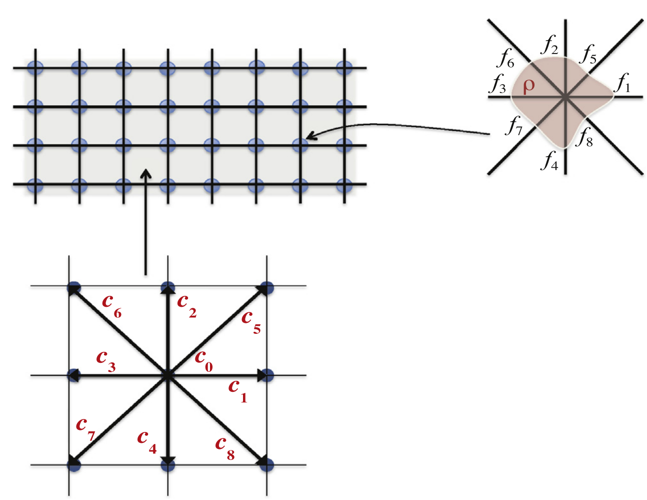

Fig. 1. D2Q9 particle speed model.