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Flow and motion characteristics of a freely falling square particle in a channel

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ABSTRACT

This study numerically simulated the motion of a square particle free falling in a two-dimensional vertical channel to investigate the effects of the off-center distance and the Reynolds number. The motion regimes were classified into non-oscillatory motion, regular oscillatory motion, and irregular oscillatory motion. The effect of the off-center distance on the particle motion became significant as the Reynolds number increased, resulting in the bifurcation of the motion regime. There was a critical off-center distance beyond which the critical Reynolds numbers corresponding to the limits of the regimes depended on the off-center distance. The mean amplitude of the transverse oscillation decreased as the ratio of the density of the particle to that of the fluid increased, and this was more significant as the off-center distance and Reynolds number increased. Moreover, as the Reynolds number increased, the drag coefficient decreased and eventually converged.

1. Introduction

Sedimentation phenomena are observed in many scientific fields such as chemical engineering [1,2], fluid mechanics [3], geology [4], and biology. Systems of free falling particles have been used in various studies and industrial applications. Examples are the petroleum industry, studies of sandstorms, river sediment resuspension and transport, mixing processes when sediment-laden rivers enter lakes, powder transport by pneumatic conveyance, fluidized beds in chemical reactors, and water treatment [5,6]. The motion of free falling particles has been studied with simplified models. Many researchers [7–18] have presented the results of their two-dimensional simulation of free falling circular particles. Moreover, three-dimensional simulations of free falling spherical particles have been conducted by several researchers [6,11,14,16,19–22].

Previous researchers have reported that the major factors affecting the motion of a free falling particle are the off-center distance, channel width, fluid properties, density ratio, inter-particle interaction, and particle shape. Hu et al. [7] reported the effect of off-center distance on the flow of a circular particle. They also observed the drafting–kissing–tumbling scenario during the sedimentation process of two circular particles. Later, Hu [9] did a quantitative analysis of the effect of the off-center distance on the fluid flow and particle motion, focusing on the interstitial flow phenomena and the rotation direction of a circular particle.

The effect of the channel width and off-center distance on free falling circular particles was reported by Feng et al. [8]. In particular, they identified five regimes in the motion of a circular particle for various Reynolds number (*Re*) ranges.

Several researchers [12,15,20,22,23] have investigated the effect of the properties of the fluid on the motion of free falling particles. They reported that the motion pattern of a particle free falling through a non-Newtonian fluid was different from that when falling through a Newtonian fluid. The effect of the density ratio on the particle motion has also been investigated with a spherical particle [23–25].

Furthermore, studies on the effect of inter-particle interaction on particle motion have been conducted by considering two-particle interaction [7,8,11–14,17,19–22] and the sedimentation of many particles [6,10–15,17–22].

Feng et al. [8], Wachs [18], and Sharma and Patankar [21] investigated the effect of particle shapes on free fall. Feng et al. [8] compared the free falling motions of circular and elliptical particles. Wachs [18] investigated the differences in the motions of free falling circular, square, and triangular particles. Furthermore, Sharma and Patankar [21] numerically simulated the free falling motions of disk and plate shaped particles.

As discernible from the foregoing, the major factors affecting the motion of a free falling particle are well established. However, the free falling motion of a square particle, which this study considers, has not been widely investigated. According to our literature review, only one paper by Wachs [18] considered the motion of a free falling square particle. Moreover, although Wachs [18] investigated

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the characteristics of the motion in a channel, he only focused on the effect of the variation in the Reynolds number for a given off-center distance. The main purpose of the present study is to investigate the effects of the off-center distance and the Reynolds number on the motion of a square particle falling freely in a channel. The particle motion regime map and the drag coefficient are presented as a function of the Reynolds number and the off-center distance. In addition, the effect of the ratio of the density of the particle to that of the fluid on the motion is discussed.

2. Computational details

2.1. Numerical methods

Many numerical schemes such as the arbitrary Lagrangian–Eulerian technique [7–10,19–21], the distributed-Lagrangian-multiplier-based fictitious-domain method [11–13,15,18,22], the immersed-boundary method [6,14,17], and the direct-forcing/fictitious-domain (DF/FD) method [5,16,26] have been used to simulate the phenomena of free falling particles. In recent years, the DF/FD method [5,16,26] has been successfully applied to analyze the free falling motion of a particle and the fluid flow around the particle. Thus, the present study adopted the DF/FD method to investigate the motion of a free falling square particle.

The continuity and momentum conservation equations that govern the incompressible viscous flow in the channel in the presence of the free falling particle are

$$\nabla \cdot \boldsymbol{u} = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho_f} \nabla p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{f}, \tag{2}$$

where $\boldsymbol{u}$ is the fluid velocity, ρ_f is the fluid density, ν is the kinematic viscosity of the fluid, and $\boldsymbol{f}$ is the volume force. A second-order accurate finite volume method was used for the spatial discretization of the governing Eqs. (1) and (2). To simulate the time advancement of the flow field, the fractional step method proposed by Choi and Moin [27] was employed.

The volume force $\boldsymbol{f}$ in Eq. (2) is used to describe the effect of a solid body on fluid flow. To calculate $\boldsymbol{f}$ in the computational domain Ω_f using the DF/FD method, Eq. (2) is transformed and discretized as follows:

$$\boldsymbol{f} = \frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \frac{1}{\rho_f} \nabla p - \nu \nabla^2 \boldsymbol{u} = \frac{\partial \boldsymbol{u}}{\partial t} + RHS \tag{3}$$

$$\boldsymbol{f} = \frac{\boldsymbol{u}^{n+1} - \boldsymbol{u}^n}{\Delta t} + RHS^{n+1/2} = \frac{\boldsymbol{u}^{n+1} - \tilde{\boldsymbol{u}}}{\Delta t} + \frac{\tilde{\boldsymbol{u}} - \boldsymbol{u}^n}{\Delta t} + RHS^{n+1/2} \tag{4}$$

where RHS represents the reorganization of the convective, pressure, and diffusion terms, and $\tilde{\boldsymbol{u}}$ is the preliminary velocity. In the discretization process, the advection terms were treated explicitly using the second-order Adams–Bashforth scheme, and the diffusion terms were treated implicitly using the second-order accurate Crank–Nicolson scheme. The definitions of the computational domain Ω_f of a fluid flow and the fictitious domain Ω_p of a particle motion using the DF/FD method are shown in Fig. 1.

The preliminary velocity $\tilde{\boldsymbol{u}}$ in Ω_f should satisfy the momentum equation as follows:

$$\frac{\tilde{\boldsymbol{u}} - \boldsymbol{u}^n}{\Delta t} + RHS^{n+1/2} = 0. \tag{5}$$

Therefore, by substituting Eq. (5) into Eq. (4), Eq. (4) can be simplified as

$$\boldsymbol{u}^{n+1} = \tilde{\boldsymbol{u}} + \boldsymbol{f} \Delta t. \tag{6}$$

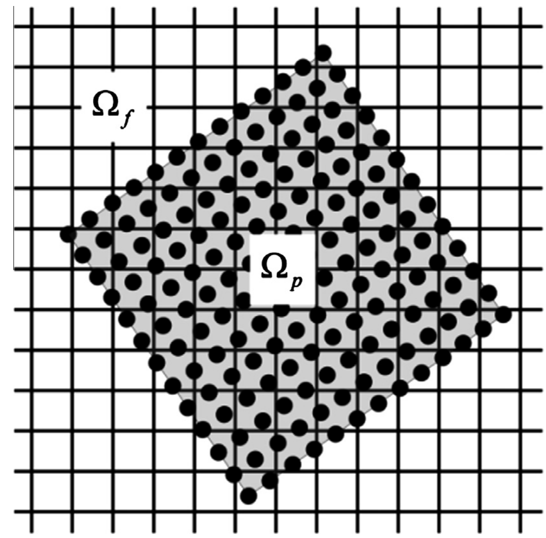

Fig. 1. Definition of computational domain Ω_f and fictitious domain Ω_p. The computational domain is represented by the line grid and the fictitious domain by the dots.

For the interaction between the fluid and solid domains, the unknown volume force $\boldsymbol{f}$ in Ω_f is calculated from the volume force $\boldsymbol{F}$ in the fictitious domain Ω_p. F is derived from the desired velocity $\boldsymbol{U^d}$ and the preliminary velocity $\tilde{\boldsymbol{U}}$ of the forcing points over Ω_p (which are distributed both inside and on the boundary of the particle) as follows:

$$\boldsymbol{F} = \frac{\boldsymbol{U}^d - \tilde{\boldsymbol{U}}}{\Delta t}, \tag{7}$$

$$\boldsymbol{U}^d = \boldsymbol{U}_c + \omega_c \times (\boldsymbol{X} - \boldsymbol{X}_c), \tag{8}$$

$$\tilde{\boldsymbol{U}} = \sum_{\boldsymbol{x} \in \Omega_f} \tilde{\boldsymbol{u}} \delta_h (\boldsymbol{x} - \boldsymbol{X}) h^2, \tag{9}$$

where the lowercase and uppercase letters represent the values in Ω_f and Ω_p, respectively. $\boldsymbol{U_c}$, ω_c, $\boldsymbol{X_c}$, and h in Eqs. (8) and (9) are the translational velocity, rotational velocity, center coordinates of the particle, and mesh size, respectively. The preliminary velocity $\tilde{\boldsymbol{U}}$ in Ω_p is hooked on that in Ω_f as shown in Eq. (9). Thus, the effect of the change in the fluid flow on the motion of the rigid body is considered in the process of analyzing the rigid-body motion. δ_h is the discrete Dirac delta function proposed by Roma et al. [28] and is expressed as

$$\delta_h(\boldsymbol{x} - \boldsymbol{X}) = \frac{1}{h^2} \phi\left(\frac{x - X}{h}\right) \phi\left(\frac{y - Y}{h}\right), \tag{10}$$

$$\phi(r) = \begin{cases} \frac{1}{6}\left(5 - 3|r| - \sqrt{-3(1 - |r|)^2 + 1}\right), & 0.5 \leqslant |r| \leqslant 1.5, \\ \frac{1}{3}\left(1 + \sqrt{-3|r|^2 + 1}\right), & |r| \leqslant 0.5, \\ 0, & \text{otherwise}. \end{cases} \tag{11}$$

The equation for the rigid-body motion is governed by Newton's equation of motion defined by Yu and Shao [16]. Thus, the translational velocity $\boldsymbol{U_c}$ and rotational velocity ω_c of the particle are calculated by the following equations:

$$V_p(\rho_p - \rho_f) \frac{d\boldsymbol{U}_c}{dt} = -\rho_f \sum_{l=1}^{N_p} \boldsymbol{F} \Delta A_l + V_p(\rho_p - \rho_f) \boldsymbol{g}, \tag{12}$$

$$I_p \frac{d\omega_c}{dt} = -\frac{\rho_p \rho_f}{\rho_p - \rho_f} \sum_{l=1}^{N_p} (\boldsymbol{X}_l - \boldsymbol{X}_c) \times \boldsymbol{F} \Delta A_l, \tag{13}$$