Computers & Fluids 79 (2013) 140–149

Contents lists available at SciVerse ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

Influence of two-dimensional smooth humps on linear and non-linear instability of a supersonic boundary layer

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ARTICLE INFO

Article history:
Received 19 December 2012
Received in revised form 28 February 2013
Accepted 27 March 2013
Available online 6 April 2013

Keywords:
Supersonic boundary layer
Stability
Hump
Parabolized stability equations
Oblique breakdown

ABSTRACT

Stability of a supersonic boundary layer over two-dimensional smooth humps with their heights considerably smaller than the local boundary layer thickness is studied by using parabolized stability equations (PSEs). Influence of humps on linear and non-linear evolution of first mode oblique wave in Mach 1.6 boundary layers is investigated. Overall destabilization influence of the hump is confirmed for both linear and non-linear cases. For the case of linear stability, an overall effect of the hump on destabilization is found to be similar to the case of subsonic boundary layer. However, in contrast to the subsonic case, considerable increase of growth rates realized in the fore part of the hump is found to contribute significantly to the overall destabilization in supersonic boundary layer. Oblique breakdown is investigated for the case of non-linear stability study. When compared to the case of flat plate, all the characteristic stages of the oblique breakdown are found to appear at a far more upstream position due to the hump. Results of parametric studies to examine the effect of hump height, location, etc. on stability characteristics are also given. Influence of dip instead of hump is also found to be very significant.

1. Introduction

One important factor affecting boundary layer transition is a surface roughness element. In practical aerodynamic configurations, many types of roughness elements such as humps, waviness, and rivets are often encountered. Since the stability and transition of boundary layer are strongly influenced by surface roughness, the effect of roughness has been studied for a long time. Some of earlier experimental studies provided empirical criteria for transition location with respect to several parameters [1]. Since several experimental studies revealed that transition over roughness is stability governed phenomena [2,3], the boundary layer stability over roughness has received much research attention. The influence of roughness on the boundary layer stability involved in a natural transition scenario was studied first [4]. It is widely known that the initial stage of natural transition process is linear modal growth of instability waves introduced into boundary layer through receptivity process [4]. Thus, theoretical studies have investigated mainly the effect of roughness on the modal growth of the instability waves [5–15].

Recent numerical studies have simulated the flow around an isolated or discrete roughness (three-dimensional) as briefly described in Choudhari et al. [16]. Evidences have been found to elucidate that instability mechanisms other than the modal instability can become significant when roughness is involved. For example, depending on the shape and size of roughness, absolute instability related to vortex shedding and wake instability might be possible mechanisms leading to transition [17]. Transient growth also has been known to be a possible instability mechanism related to transition due to distributed roughness [18]. In the presence of roughness elements, we thus see that various instability mechanisms and complex interactions among those can usually be involved in the transition process.

For modal growth of instability wave over a roughness element, most of the previous theoretical studies were based on the linear stability theory (LST) [19] and studies so far were limited to the case of two-dimensional smooth roughness element. Many LST studies including parametric study were carried out to investigate the stability of boundary layer over a smooth hump and backward- or forward-facing step [5–10]. The effects of hump height, length, location, shape, unit Reynolds number, free-stream Mach number, etc. on the predicted transition location based on e^N criterion were also evaluated [11,12]. Wie and Malik [13] and Gao et al. [14] carried out PSE analysis which can account for flow non-parallelism and surface curvature to study the linear stability over surface waviness and a smooth hump, respectively. Destabilizing effect of roughness on the modal growth of the instability waves was confirmed. Wörner et al. [15] also confirmed the destabilization effect by direct numerical simulation (DNS) analysis.

Most of the previous theoretical studies have dealt with the linear stability of subsonic boundary layer over a smooth roughness.

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0045-7930/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compfluid.2013.03.018

Non-linear stability and/or supersonic boundary layer stability have rarely been studied so far. Recently, present authors [20] theoretically investigated non-linear evolution of instability waves in incompressible boundary layer over a smooth hump by using non-linear parabolized stability equations (PSEs) analysis. Although linear and non-linear stability of boundary layer has been studied recently by using DNS for simple configurations, investigations on the non-linear stability of boundary layer over roughness by DNS are rarely found within the framework of stability study.

In this study, linear and non-linear stability of supersonic boundary layer over a roughness element is investigated based on parabolized stability equations (PSEs) analysis. The influence of roughness on the evolution of instability waves of discrete mode is examined. To focus on convective instability only, a two-dimensional smooth hump with its height smaller than the local boundary layer thickness is chosen as roughness element. The PSE methodology previously used in the study for incompressible boundary layer [20] is extended to the present study of supersonic boundary layer. Mean flow data is obtained from the solution of Parabolized Navier–Stokes (PNS) equations. Effect of hump on the linear evolution of first mode oblique wave is briefly examined. For the case of non-linear stability, oblique breakdown is investigated by non-linear PSE. Influence of hump height and location is discussed. Influence of dip instead of hump is also briefly examined.

2. Method of analysis

Fig. 1 shows the schematic of the hump geometry used in the present study. L^*, b^* and h^* represent respectively the distance from the leading edge of flat plate to the location of the hump center, half width, and height of the hump. The superscript $*$ is used to denote dimensional variables. The Reynolds number Re is defined as $U^*_\infty L^*/\nu^*$ where U^*_∞ is the free-stream velocity.

The geometry of the hump is given by Eq. (1), which is the same with that used in previous studies [5,11,20].

$$y = y^*/L^* = (h^*/L^*)f(t) = hf(t) \tag{1}$$

where

$$t = (x^* - L^*)/b^* = 2(x-1)/b$$

$$f(t) = \begin{cases} ll1 - 3t^2 + 2|t|^3, & \text{if } |t| \leqslant 1 \\ 0, & \text{if } |t| > 1 \end{cases}$$

The 2-D mean flow data of supersonic boundary layer over a hump are obtained by solving PNS equations [21]. The accuracy of PNS solution as basic flow for linear stability analysis of axisymmetric flows was verified by Hejranfar et al. [22]. The non-dimensional governing equations for 2-D compressible flow in conservation-law form are written as

$$\frac{\partial \vec{U}}{\partial t} + \frac{\partial}{\partial x}(\vec{E}_i - \vec{E}_v) + \frac{\partial}{\partial y}(\vec{F}_i - \vec{F}_v) = 0 \tag{2}$$

where $\vec{U}$ is the flow variable vector, $\vec{E}$ and $\vec{F}$ the flux vector in x and y direction, respectively. The subscripts i and v represent inviscid and viscous flux. Application of curvilinear computational coordinate grid transformation and dropping the streamwise (ξ-direction) derivatives of viscous terms yields PNS which can be compactly expressed as

$$\frac{\partial \vec{U}}{\partial t} + \frac{\partial}{\partial \xi}(\overline{E}_i) + \frac{\partial}{\partial \eta}(\overline{F}_i - \overline{F}_v) = 0 \tag{3}$$

where

$$\overline{E}_i = \frac{1}{J}(\xi_x \vec{E}_i + \xi_y \vec{F}_i), \quad \overline{E}_v = 0, \; \overline{F}_i = \frac{1}{J}(\eta_x \vec{E}_i + \eta_y \vec{F}_i),$$
$$\overline{F}_v = \frac{1}{J}(\eta_x \vec{E}'_v + \eta_y \vec{F}'_v)$$

The primed terms in the above denote the terms in which streamwise viscous terms are dropped. Details of the above equations are readily available in Refs. [21,23]. In this study, the time iterative PNS (TIPNS) scheme of Tannehill et al. [21] is employed. The Steger–Warming splitting scheme [24] is used to split the streamwise flux vector. No-slip and adiabatic wall boundary condition is specified.

A computer code for PNS solution for both 2-D and axisymmetric flows was made and was validated through several typical axisymmetric flow cases given in Refs. [21,23]. Since the iterative PNS scheme has been proved to capture the upstream influence and inviscid–viscous interaction properly, we believe that the boundary layer over a smooth hump with small height can be computed with sufficient accuracy by the PNS scheme. The similarity solution of compressible boundary layer equation is imposed as inflow condition at far upstream of the hump for PNS marching. The similarity solution was obtained from the boundary layer code which is 4th order accurate in surface normal direction following the work of Iyer [25].

As is well known, PSE can take into account the flow non-parallelism and curvature effect. The PSE approach is computationally more efficient than DNS and requires even less computational time than LST since it performs the stability analysis by downstream marching from the initial condition. Further, it also enables us to carry out non-linear stability study. The PSE has widely been used to study linear and non-linear stability of subsonic and supersonic boundary layers over simple geometries. The PSE formulation and solution technique can be found in many Refs. [26–28].

The PSE method of the present study is the same with that of our previous work for incompressible boundary layers over a hump [20]. For brevity, we only give final form of the equations and more details on formulation is referred to Ref. [20]. The PSE can compactly be written as follows:

$$\hat{D}_{mn}\psi_{mn} + \hat{A}_{mn}\frac{\partial \psi_{mn}}{\partial x_1} + \hat{B}_{mn}\frac{\partial \psi_{mn}}{\partial x_2} = \hat{V}_{22,mn}\frac{\partial^2 \psi_{mn}}{\partial x_2^2} + \frac{F_{mn}}{A_{mn}} \tag{4}$$

where

$$\hat{D}_{mn} = -i\omega\overline{\Gamma} + i\alpha_{mn}\overline{A} + in\beta\overline{C} + \overline{D} + \alpha_{mn}^2\overline{V}_{11} + n^2\beta^2\overline{V}_{33} + n\alpha_{mn}\beta\overline{V}_{31} - i\frac{d\alpha_{mn}}{dx_1}\overline{V}_{11}$$

$$\hat{A}_{mn} = \overline{A} - i2\alpha_{mn}\overline{V}_{11} - in\beta\overline{V}_{31} \quad \hat{B}_{mn} = \overline{B} - i\alpha_{mn}\overline{V}_{12} - in\beta\overline{V}_{23}$$

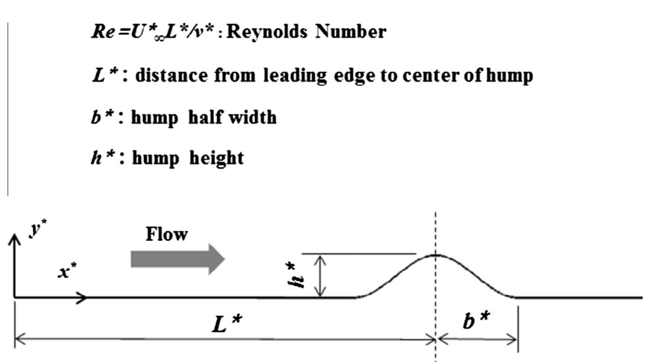

Fig. 1. Schematic of hump geometry.