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A study of various factors affecting Newtonian extrudate swell

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ABSTRACT

Finite-element simulations have been undertaken for the benchmark problem of extrudate swell present in extrusion. Both cases of planar and axisymmetric domains were considered under laminar, isothermal, steady-state conditions for Newtonian fluids. The effects of inertia, gravity, compressibility, pressure-dependence of the viscosity, slip at the wall, and surface tension are all considered *individually* in parametric studies covering a wide range of the relevant parameters. The present results extend previous ones regarding the shape of the extrudate and in particular the swelling ratio. In addition, the excess pressure losses in the system (exit correction) were computed. The effect of the domain length is also studied and is found to be of importance in all cases, except for slip and surface tension effects. The effect of the extrudate length is particularly important for inertia and gravity flows. Inertia reduces the swelling down to the asymptotic theoretical values at infinite Reynolds numbers. Gravity acting in the direction of flow also reduces exponentially the swelling. When the flow is creeping and gravity is zero, surface tension, slip at the wall, and pressure-dependence of viscosity, all decrease the swelling monotonically, while compressibility increases it after a small initial reduction. The exit correction decreases monotonically with inertia, gravity, and slip, increases monotonically with compressibility and pressure-dependence of the viscosity, and is not affected by surface tension.

1. Introduction

Extrudate swell (“die swell”) is a well-known phenomenon exhibited by viscous fluids exiting long slits or capillary dies [1]. Within the context of non-Newtonian fluid mechanics, this type of flow is of interest in polymer processing, and in particular in the flow of polymer melts in extrusion [2]. Numerical solutions of the extrudate-swell problem were provided in the mid-1970s by a number of researchers [3–5], starting with the pioneering work of Tanner [3], who for the first time calculated correctly the extrudate position. These works dealt primarily with Newtonian fluids and showed how the extrudate surface develops under various conditions, in agreement with experiments [3–6]. The 1980s and 1990s saw a major effort to calculate extrudate swell with viscoelastic models, and these efforts are summarized in various review papers and monographs [1,7–9].

Although the problem is well understood from the physics and fluid mechanics points of view, it has become evident from available numerical simulations that the flow changes considerably when using different constitutive equations or domain geometry (planar vs. axisymmetric). Changing the constitutive equation may lead to a flow that is dramatically different, in very interesting and unpredictable ways [10–14]. The same is true for other parameters influencing the fluid mechanics of extrudate swell flow, ranging from inertia [6,15,16], to gravity [17], to surface tension [6,15,18,19], etc.

A key work by Georgiou et al. [20], which appeared as a short note, showed both computationally and in comparison with experiments that inertia, gravity and surface tension have a pronounced effect on the extrudate shape, reducing it appreciably when gravity acts in the flow direction. Subsequently, Georgiou and co-workers [21–25] have addressed the influence of some standard fluid mechanics parameters on extrudate swell, but again not in full parametric studies. Furthermore, the discussion of pressure results, and hence the excess pressure losses associated with exit flow, which are an integral and important part of the solution, have been neglected.

It is, therefore, the purpose of the present paper to revisit the steady-state Newtonian extrudate-swell problem in both planar and axisymmetric geometries for a full parametric study of the effects of inertia, gravity, compressibility, a pressure-dependent viscosity, slip at the wall, and surface tension on the free surface. The range of parameters will be from the base case of creeping flow without any other effects to the other extreme dictated either from physical arguments or loss of convergence. The emphasis will be

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on providing detailed results both for the free surface location (extrudate swell) and the excess pressure losses in the system (exit correction) as a function of the relevant fluid mechanics dimensionless parameters, as it was done recently for the benchmark fountain flow problem in injection moulding [26].

2. Mathematical modelling

2.1. Governing equations

The geometry of the axisymmetric extrudate-swell problem is shown schematically in Fig. 1, along with the boundary conditions. Cylindrical coordinates are the natural choice, and the gravitational acceleration vector, $\bar{g}$, is assumed to be in the direction of flow. Moreover, the flow is assumed to be isothermal and steady-state [1,27,28]. The flow is governed by the continuity and momentum equations:

$$\nabla \cdot (\rho \bar{u}) = 0, \tag{1}$$

$$\rho \bar{u} \cdot \nabla \bar{u} = -\nabla p + \nabla \cdot \bar{\bar{\tau}} + \rho \bar{g}, \tag{2}$$

where ρ is the density, $\bar{u}$ is the velocity vector, p is the pressure, and $\bar{\bar{\tau}}$ is the extra-stress tensor. Assuming that the fluid is dense with a zero dilatational (bulk) viscosity [1,2], the viscous stress tensor for a compressible Newtonian fluid is given by:

$$\bar{\bar{\tau}} = \mu(\nabla \bar{u} + \nabla \bar{u}^T) - \frac{2\mu}{3}(\nabla \cdot \bar{u})\bar{\bar{I}}, \tag{3}$$

where μ is the viscosity and $\bar{\bar{I}}$ is the unit tensor. Both the density and the viscosity are assumed to be pressure-dependent. The following linear equation of state is considered [29]:

$$\rho = \rho_0[1 + \beta(p - p_0)], \tag{4}$$

where β is the isothermal compressibility assumed to be constant, and ρ_0 is the density at the reference pressure p_0.

Similarly, the viscosity can be a function of pressure, either linear or exponential [30,31]. In the present work, the latter form is employed:

$$\mu = \mu_0 \exp[\beta_p(p - p_0)], \tag{5}$$

where β_p is the constant pressure-shift coefficient, and μ_0 is the viscosity at the reference pressure p_0.

The constitutive equation for Newtonian fluids (Eq. (3)) is substituted into the momentum equations (Eq. (2)), and the equation of state (Eq. (4)) into both the continuity (Eq. (1)) and momentum equations. The resulting system of partial differential equations is closed by appropriate boundary conditions.

2.2. Boundary conditions

As already mentioned, the solution domain and boundary conditions for the axisymmetric geometry are shown in Fig. 1. The boundary conditions are as follows:

(a) Along the axis of symmetry AB, we take the standard symmetry conditions of zero radial velocity and shear stress ($u_r = 0$, $\tau_{rz} = 0$).
(b) Along the wall DS we assume that the normal velocity is zero (no penetration) and that the tangential velocity obeys a linear slip equation [21,32], i.e.,

$$\bar{n} \cdot \bar{u} = 0, \quad \bar{t} \cdot \bar{u} = \beta_{sl}(\bar{t}\bar{n} : \bar{\bar{\tau}}), \tag{6}$$

where β_{sl} is the slip coefficient, and $\bar{n}$ and $\bar{t}$ are the normal and tangential unit vectors to the wall. For straight walls used here, these conditions translate to the radial velocity being zero ($u_r = 0$) and the axial velocity being proportional to the wall shear stress τ_w ($u_z = \beta_{sl}\tau_w$). It should be noted that the no-slip case ($u_z = u_r = 0$) is recovered as β_{sl} goes to zero.

(c) Along the free surface SC (becoming SC′) the kinematic condition $\bar{n} \cdot \bar{u} = 0$ ensures that the free surface is a streamline. Moreover, the tangential stresses vanish ($(\bar{\bar{\sigma}} \cdot \bar{n}) \cdot \bar{t} = 0$), while the normal stresses satisfy a force equilibrium according to [23–25]:

$$(\bar{\bar{\sigma}} \cdot \bar{n}) \cdot \bar{n} = -2R_c\gamma - p_0, \tag{7}$$

where $\bar{\bar{\sigma}} = -p\bar{\bar{I}} + \bar{\bar{\tau}}$ is the total stress, γ is the surface tension, p_0 is the reference pressure (set to 0), and $2R_c$ is the mean curvature of the free surface given by [23–25]:

$$-2R_c = \frac{h_{zz}}{[1 + h_z^2]^{3/2}} - \frac{\alpha}{r\sqrt{1 + h_z^2}}. \tag{8}$$

In the above, the subscripts z and zz denote first- and second-order differentiation of the free surface location h with respect to z, and r is the local radius. The parameter α is an auxiliary one, being 0 for planar flows and 1 for axisymmetric flows. Thus, the second term is 0 in planar flows. It is also clear that in the case of zero surface tension, the normal stress on the free surface vanishes.

(d) Along the outflow plane BC (becoming BC′), taken sufficiently far downstream from the exit so that the flow is uniform, the radial velocity is zero ($u_r = 0$) and the normal stress is given by

$$\sigma_{zz} = -\frac{\alpha\gamma}{h_f}, \tag{9}$$

where h_f is the final radius at the outlet (distance BC′). Note that the normal stress in the case of planar flow is zero (i.e., the surface tension has no effect on the normal stress on the outflow plane).

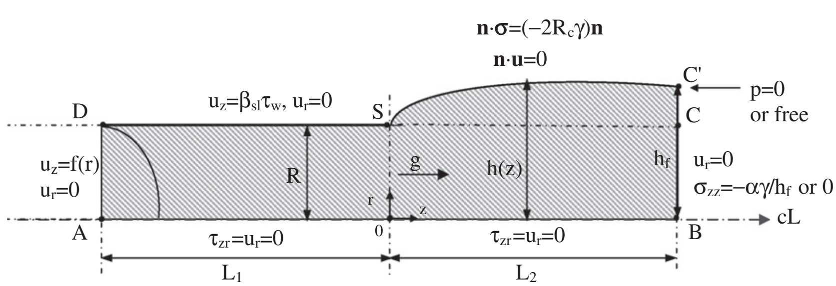

Fig. 1. Schematic diagram of flow domain and boundary conditions for extrusion flow from a die and the accompanying phenomenon of extrudate swell. The constant $\alpha = 0$ for the planar case and $\alpha = 1$ for the axisymmetric one.