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Approximate ballistics formulas for spherical pellets in free flight

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ABSTRACT

The ballistics equations for spherical pellets in free flight are simplified through appropriate scaling of the pellet velocity and pellet distance. Two different drag coefficient curves are averaged to yield a single curve applicable to shot pellets and round balls. The resulting S-shaped drag coefficient curve is approximated by three straight-line segments. The scaled ballistics equations are then solved exactly and simple formulas are found for the velocity and flight time with respect to trajectory distance. The formulas are applicable to spherical shot pellets and round balls of any composition under any atmospheric conditions. The formulas are amenable to quick and easy computation and may also serve as an aid in understanding and comparing black-box ballistics calculators. For shotshell ballistics, an important assumption in the present investigation is that the pellets are moving as single, free spheres and not as a dense cloud or in a shot column, in particular, the pellets are not interacting during flight. Therefore, the formulas are most appropriate for single round balls, for large shot sizes, and for pellets of small shot size fired from open chokes. The formulas are clear and accessible, and can be implemented by military or law enforcement personnel as well as hunters and shooters. This work differs from previous investigations in that accurate ballistics formulas are derived for spherical projectiles of shotguns and muzzleloaders using realistic drag coefficients.

1. Introduction

In the present investigation, approximate ballistics formulas are derived for spherical projectiles of shotguns or muzzleloaders using realistic drag coefficients. To derive the ballistics formulas, the differential equations are simplified using dimensionless variables for velocity and distance and transforming the problem into one for velocity versus distance rather than for velocity versus time. The drag coefficient is accurately approximated by a continuous piecewise linear function that depends solely on Mach number. Using this drag coefficient, the ballistics equations are solved exactly resulting in analytical expressions for velocity and flight time versus distance. This work is useful as, currently, accurate ballistics formulas for shotguns and muzzleloaders are not known analytically and ballistics curves must be calculated computationally. An important assumption is that the pellets are not interacting during flight. Therefore, the formulas are most appropriate for large shot sizes and open chokes. The presentation is complete and self-contained and the formulas can be readily implemented.

Three forces are important in shot pellet dynamics. A major force due to the drag of the air acts in the opposite direction of the motion of the pellet. Two minor forces involve the downward force of gravity and a sideways force due to a crosswind that acts perpendicular to pellet travel. (The component of the wind in the direction of pellet travel is small compared with the pellet's muzzle velocity and is generally neglected.) The drag on the pellet is much larger than gravity or crosswind forces and, as a result, the flight dynamics with drag can be first calculated and then corrected for gravitational drop and wind drift [19,26].

Newton's second law for a pellet undergoing drag has the form

$$ma = -F \tag{1}$$

where m is the pellet mass, a is the acceleration, and F is the drag force on the shot pellet (See Table 1 for notation). The drag force F increases with velocity and with pellet diameter. The drag on single spherical objects in air has been studied experimentally and theoretically in several investigations, e.g., [3,9,10,14,18,21,23,24]. The drag force F depends on velocity, Reynolds number, and the Mach number. For spherical pellets, the drag force is equal to [4]:

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Table 1
Notation used in the paper (length units are all cm to simplify formulas).

Notation	Units	Description
M	unitless	Mach number, i.e., dimensionless velocity $M = v/v_s$
R_e	unitless	Reynolds number, i.e., dimensionless velocity $R_e = D\rho_a v/\mu_a$
Z	unitless	dimensionless distance $z = \rho_a x/(\rho_p D) = x/k_z$
k_z	cm	scale factor in distance $k_z = D\rho_p/\rho_a$
t	sec	pellet flight time
x	cm	pellet position, i.e., trajectory distance
v	cm/sec	pellet velocity
v_0	cm/sec	pellet muzzle velocity, corresponding Mach number is $M_0 = v_0/v_s$
a	cm/sec^2	pellet acceleration
m	gm	pellet mass
D	cm	pellet diameter
ρ_p	gm/cm^3	pellet density
v_s	cm/sec	speed of sound in air
ρ_a	gm/cm^3	air density
μ_a	Gm/(cm sec)	air viscosity
T_a	$^\circ C$	air temperature
P_a	Inches Hg	air pressure
RH	Unitless	relative humidity of the air ($0 < RH < 1$)
F	gm cm/sec^2	drag force
$C(M)$	unitless	drag coefficient which depends on M
g	cm/sec^2	gravitational acceleration $g = 980.66$ cm/sec^2
v_w	mph	crosswind velocity
P_V	inches Hg	vapor pressure
R_g	inches Hg cm^3 /($^\circ C$ gm)	gas constant $R_g = 84.763$(inches Hg cm^3)/($^\circ C$ gm)

$$F = \pi D^2 \rho_a v^2 C(M, R_e)/8 \tag{2}$$

where D is pellet diameter, ρ_a is air density, v is pellet velocity, and $C(M, R_e)$ is the drag coefficient. The drag coefficient, $C(M, R_e)$, is a measure of the pellet's resistance in air and depends in a complicated way on the Reynolds number R_e and the Mach number M for a compressible fluid such as air [3,18,21,22]. The Mach number is equal to the pellet velocity divided by the speed of sound in air and the Reynolds number is equal to the product of the pellet diameter, air density, and pellet velocity divided by the air viscosity. The Mach and Reynolds numbers are considered in greater detail in the next section. In particular, though, under constant atmospheric conditions and a given pellet diameter, the Mach and Reynolds numbers are proportional to pellet velocity.

By equations (1) and (2), the pellet velocity satisfies the differential equation

$$m\frac{dv}{dt} = -\pi D^2 \rho_a v^2 C(M, R_e)/8. \tag{3}$$

Equation (3) is the principal differential equation for the exterior ballistics of spherical pellets. The solution of equation (3) gives the pellet velocity with flight time and, by integration, the trajectory distance with flight time. Then, corrections due to gravity or crosswind drift can be applied. If atmospheric conditions are constant and the pellet size and composition are fixed, then R_e and M are proportional to pellet velocity and equation (3) can be written as

$$\frac{dv}{dt} = -kv^2 G(v) \tag{4}$$

where k is a constant and G is a function which depends on pellet velocity v.

Due to the complicated nature of the drag coefficient as a function of Mach number and Reynolds number, equation (3) or (4) cannot be solved exactly. However, in order to understand the nature of pellet ballistics, simple approximations have been made for the drag coefficient so that analytical solutions to (3) or (4) can be determined and studied. For example, it is sometimes assumed that $G(v) = cv^i$ where c is a constant and $i = 0$ or $i = 1$ [1,11]. The resulting solutions yield insight into the ballistics dynamics but are not highly accurate for the entire range of pellet velocities in a typical trajectory.

If the drag coefficient is known in a functional and/or tabulated form, then (3) can be accurately solved in a computational manner. Some computational methods successively solve the differential equation in small time steps for pellet velocity and position. A classic ballistics program was developed by E. D. Lowry and used by J. Taylor to generate numerous shotshell ballistics tables [26]. Several ballistics programs currently available online include: “Shotgun Load Comparison Calculator Program” of Westslope Magazine [30], “Shotgun Simulator” of Blackbart Software [5], “Shogun Ballistics” of Connecticut Muzzleloaders [12], and “Hornady Ballistic Calculators” of Team Hornady [17]. One disadvantage of these computer programs is that the drag coefficients used in the programs are, in general, not clearly described.

In the present investigation, the ballistics equations for velocity and flight time are solved exactly using an accurate approximation to the drag coefficient. In order to accomplish this, equation (3) is first redefined as a problem involving dimensionless velocity as a function of dimensionless distance rather than velocity as a function of time. The simpler forms of the equation and its solution motivate this change. Next, by considering drag coefficients for different Reynolds numbers and for pellets of different sizes, it is explained how the drag coefficient can be accurately approximated by a continuous piecewise linear function that depends solely on Mach number. Using this drag coefficient, the ballistics equations are solved exactly resulting in analytical formulas for pellet velocity and flight time with trajectory distance. The derived ballistics formulas are tested and several examples are described. In addition, for completeness, relevant notes are given on corrections for gravitational drop and wind drift, on the estimation of certain physical parameters of air, and on the effects of shot clouds and string length.