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A Quaternion-Based LOS Guidance Scheme for Path Following of AUVs

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Abstract: This paper presents a quaternion version of the well-known Line-of-Sight (LOS) guidance algorithm for marine applications. The transformation from Euler angles is achieved by exploiting the nature of the quaternion structure and using fundamental half-angle formulae from trigonometry. First, the Euler angles version of the LOS guidance algorithm is briefly presented for two uncoupled cases: a) the horizontal xy-plane, and b) the vertical zx-plane. Then, a coupled case is also considered and the transformation procedure is presented for all three cases. The vehicle considered pertains to a 5-DOF kinematics model where the roll angle is neglected, typical of torpedo-shaped Autonomous Underwater Vehicles (AUVs). Naturally, the Euler angles representation of the system involves singularities which, in the general 3-D space navigation case, should be avoided. The presented method aims at providing a singularity-free and computationally-efficient version of the conventional LOS algorithm.

Keywords: Autonomous underwater vehicles, path following, LOS guidance, quaternions

1. INTRODUCTION

The Line-of-Sight (LOS) algorithm is a well-documented guidance method in both marine and aerial vehicles applications, see for instance Yanushevsky (2011), Breivik (2010), Breivik and Fossen (2009). Its main advantages are its simplicity and efficiency in generating appropriate heading reference trajectories such that, when tracked by the heading autopilot, they guarantee the vehicle's smooth convergence to the desired path. The method is based on a simple geometry which, when it comes to marine applications, refers to the LOS vector starting at the vehicle's position and passing through a point $\mathbf{p}(x_{\text{los}}, y_{\text{los}})$, which is located on the path-tangential line at a lookahead distance $\Delta_h > 0$ ahead of the direct projection of the vehicle's position $\mathbf{p}(x, y)$ on to the path, this can be seen in Fig. 1. The vehicle is then assigned to reach the constantly moving point $\mathbf{p}(x_{\text{los}}, y_{\text{los}})$ and this induces the desired steering behavior.

The performance of the algorithm is affected by factors such as the value of the lookahead distance, as well as the convergence speed of the heading autopilot. Regarding the lookahead distance, large values will result in a smooth convergence, without oscillations around the desired path but also more time-consuming. On the other hand, small values will drive the vehicle faster on the desired path, but this will usually lead to an oscillatory behavior. For this reason, several variable lookahead distance methods have been proposed in the past (Pavlov et al., 2009; Oh and Sun, 2010; Lekkas and Fossen, 2012). The issue of the heading autopilot convergence is usually studied by using cascaded systems theory. In that context, the subsystem consisting of the heading autopilot and the vehicle is considered to be a perturbation to the subsystem consisting of the LOS guidance and the vehicle, for more information the interested reader is referred to Børhaug and Pettersen (2005), Lekkas and Fossen (2012).

It is a well known fact that, in the general case, the Euler angles representation of the 6-DOF kinematics involves singularities for the pitch angles $\theta = \pm 90^\circ$ (Fossen, 2011). Consequently, it is useful to derive a quaternion version of the conventional LOS guidance for AUVs. Moreover, the quaternion representation is more computationally efficient compared to Euler angles since it does not include trigonometric functions. This makes it even more suitable for applications involving unmanned vehicles where the on-board computational power might be more limited. This paper serves as the first step toward this direction. The quaternion representation of the LOS algorithm is derived for two uncoupled 3-DOF cases: a) the horizontal xy-plane, and b) the vertical zx-plane, and for a coupled case where the sideslip angle is also a function of the vertical motion. For each case, the terms of the guidance law are transformed from Euler angles to quaternion by taking into account the nature of quaternions that correspond to rotations and using simple trigonometric identities.

The rest of this paper is organized as follows: Section 2 presents the vehicle model considered. In Section 3, the Euler representation of the LOS guidance algorithm is presented for the uncoupled horizontal and vertical planes

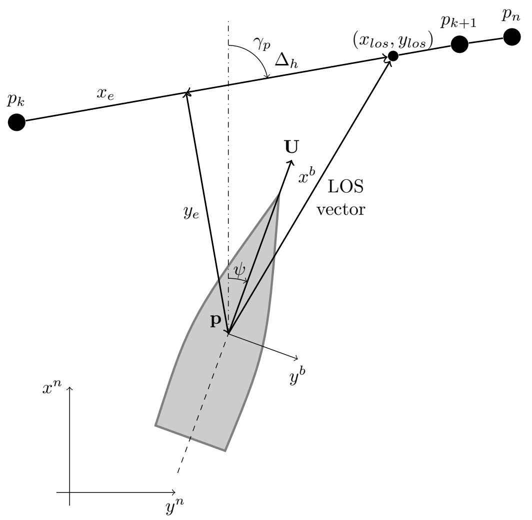

Fig. 1. Line-of-sight guidance geometry for straight lines in the xy plane. Here the sideslip angle is equal to zero.

as well as the coupled case. The quaternion transformation for the uncoupled horizontal plane is given in Section 4, for the uncoupled vertical plane in Section 5 and for the coupled case in Section 6. Some simulation results can be found in Section 7, and Section 8 concludes the paper.

2. VEHICLE MODEL

2.1 Vehicle Dynamics

Following the methodology of Børhaug and Pettersen (2005), for the path-following task we can neglect the roll angle, hence for an underactuated autonomous vehicle the following 5-DOF dynamic model can be used:

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu}, \tag{1}$$

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}, \tag{2}$$

where $\mathbf{M}$ is the mass and inertia matrix, $\mathbf{C}(\boldsymbol{\nu})$ is the Coriolis and centripetal matrix, $\mathbf{D}(\boldsymbol{\nu})$ is the damping matrix, $\mathbf{g}(\boldsymbol{\eta})$ describes the gravitational and buoyancy forces, and $\boldsymbol{\tau}$ includes the control forces and moments.

Accordingly, the generalized position and velocity are recognized as:

$$\boldsymbol{\eta} = (x,\ y,\ z,\ \theta,\ \psi)^{\mathrm{T}},\ \ \boldsymbol{\nu} = (u,\ v,\ w,\ q,\ r)^{\mathrm{T}}, \tag{3}$$

where (x, y, z) is the vehicle's inertial position in Cartesian coordinates, θ is the pitch angle and ψ is the yaw angle. In addition, u is the surge velocity, v is the sway velocity, w is the heave velocity, q is the pitch rate and r is the yaw rate.

2.2 Vehicle Kinematics

The model considers only absolute velocities and is the following, see Børhaug and Pettersen (2005):

$$\dot{x} = u\cos(\psi)\cos(\theta) - v\sin(\psi) + w\cos(\psi)\sin(\theta), \tag{4}$$

$$\dot{y} = u\sin(\psi)\cos(\theta) + v\cos(\psi) + w\sin(\psi)\sin(\theta), \tag{5}$$

$$\dot{z} = -u\sin(\theta) + w\cos(\theta), \tag{6}$$

$$\dot{\theta} = q, \tag{7}$$

$$\dot{\psi} = \frac{1}{\cos(\theta)}r, \quad \cos(\theta) \neq 0. \tag{8}$$

3. EULER REPRESENTATION OF THE LOS GUIDANCE LAW

3.1 Horizontal Plane LOS Guidance

In the case of decoupled horizontal plane path-following we assume that $\theta = 0°$, consequently the kinematics equation to be considered is:

$$\dot{x} = u\cos(\psi) - v\sin(\psi), \tag{9}$$

$$\dot{y} = u\sin(\psi) + v\cos(\psi), \tag{10}$$

$$\dot{\psi} = r. \tag{11}$$

The horizontal-plane speed U_h is given by:

$$U_h := \sqrt{u^2 + v^2}, \tag{12}$$

and is assumed to be positive and bounded:

$$U_{h,\min} \leq U_h \leq U_{h,\max}, \ \ 0 < U_{h,\min}. \tag{13}$$

From (12)–(13) it is implied that the vessel always has at least a nonzero surge speed. The reason for setting a minimum positive speed $U_{h,\min}$ is related to the stability proof of the LOS algorithm and a more rigorous approach is outside the scope of this paper and can be found in Lekkas and Fossen (2013). The model (9)–(11) includes only absolute velocities and describes the motion of an underactuated vehicle since only two out of three DOF's can be controlled independently, namely the yaw angle and the surge velocity.

Path Following Objective: Assuming that the vehicle is assigned to converge to the line connecting the waypoints WP_k–WP_{k+1}, the along-track and the cross-track error for a given vehicle position (x, y) are given by:

$$\begin{bmatrix} x_e \\ y_e \end{bmatrix} = \mathbf{R}^{\top}(\gamma_p) \begin{bmatrix} x - x_k \\ y - y_k \end{bmatrix}, \tag{14}$$

where (x_k, y_k) is the position of the k-th waypoint expressed in the NED frame, and the rotation matrix from the inertial frame to the path-fixed reference frame is given by:

$$\mathbf{R}(\gamma_p) = \begin{bmatrix} \cos(\gamma_p) & -\sin(\gamma_p) \\ \sin(\gamma_p) & \cos(\gamma_p) \end{bmatrix} \in SO(2). \tag{15}$$

Moreover,

$$x_e = (x - x_k)\cos(\gamma_p) + (y - y_k)\sin(\gamma_p), \tag{16}$$

$$y_e = -(x - x_k)\sin(\gamma_p) + (y - y_k)\cos(\gamma_p), \tag{17}$$

where γ_p is the horizontal-plane path-tangential angle:

$$\gamma_p = \mathrm{atan2}(y_{k+1} - y_k, x_{k+1} - x_k), \tag{18}$$

Then, the associated control objective for horizontal plane straight-line path-following is:

$$\lim_{t \to +\infty} y_e(t) = 0. \tag{19}$$

Note that the along-track error x_e does not need to be minimized in a path-following scenario, the contrary is true for applications that impose temporal constraints. The lookahead-based guidance law is given by (see Breivik and Fossen (2009)):

$$\psi_d = \gamma_p + \arctan\left(\frac{-y_e}{\Delta_h}\right). \tag{20}$$

In the presence of external disturbances, or during turns, the heading angle ψ_d and the course angle χ_d are not aligned anymore and are related in the following way:

$$\chi_d = \psi_d + \beta, \tag{21}$$