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Performance comparison of exponential, Lambert *W* function and Special Trans function based single diode solar cell models

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ABSTRACT

Accurate modeling plays an important role in solar cell simulation. In order to reveal the applicability and superiority of Special Trans function based single diode model (SBSDM), this paper presents a comprehensive comparison of SBSDM, Lambert *W* function based single diode model (LBSDM) and exponential-type single diode model (SDM). The performance difference of SBSDM, LBSDM and SDM is verified and compared in two aspects: (1) different fitness to the measured *I–V* data of solar cells and (2) different parameter extraction performance. To be objective and reproducible, the reported parameter values of standard datasets and measured datasets are employed to validate the fitness difference of the three models. The comparison results indicate that SBSDM always exhibits better fitness than LBSDM and SDM in representing the *I–V* characteristics of various solar cells and can provide a closer prediction to actual maximum power points. With the help of a ranking based branch selection strategy, a modified Nelder-Mead simplex (MNMS) algorithm is proposed to test the parameter extraction performance of SBSDM, LBSDM and SDM. The comparison results reveal that the time computational efficiency of SBSDM is inferior to SDM but superior to LBSDM. SBSDM always achieves superior accuracy and convergence speed than LBSDM and SDM, although lacking enough statistical robustness. Due to these superiorities, SBSDM is quite promising and envisaged to be the most valuable model for solar cell parameter extraction and PV system simulation.

1. Introduction

As a promising renewable energy technology, solar cell has been extensively investigated due to its unique advantage and great potential to meet the increasing energy demands. Along with the evolution of PV technology, accurate modeling and parameter extraction for closely representing the nonlinear *I–V* (current vs. voltage) characteristics have attracted considerable attention in performance evaluation of solar cell and maximum energy harvesting of PV systems [1–4].

1.1. Brief overview of three models

Over the past years, various models have been developed to describe the electrical behavior of solar cells. In practice, the exponential-type single diode model (SDM) is undoubtedly the most widely used model due to its good compromise between simplicity and accuracy [3–7].

In the equivalent circuit of SDM illustrated in Fig. 1, the *I–V* relationship of a solar cell under illumination can be formulated by the following SDM Eq. (1).

$$I = I_{ph} - I_0\left[\exp\left(\frac{V+IR_s}{nV_{th}}\right) - 1\right] - \frac{V+IR_s}{R_{sh}} \tag{1}$$

where I, V, I_{ph}, I_0, n, R_s and R_{sh} are the terminal current, terminal voltage, photocurrent, diode saturation current, diode ideality factor, series and shunt resistances respectively. Thermal voltage $V_{th} = N_s kT/q$, where N_s is the number of cells in series, $k = 1.3806503 \times 10^{-23}$ J/K is the Boltzmann constant, $q = 1.60217646 \times 10^{-19}$ C is the electronic charge, and T is the absolute temperature in Kelvin which can be calculated by 273.15 plus the cell temperature in Celsius.

It is easy to see from Eq. (1) that there are five parameters (I_{ph}, I_0, n, R_s and R_{sh}) in SDM to be extracted as accurately as possible. These parameters are not only of vital importance for performance evaluation and quality control of solar cell, but also play a significant role in maximum power point (MPP) tracking of PV systems [7–10]. For instance, the diode ideality factor n is a real quality index indicating how closely a real solar cell follows the ideal cell, and its value depends critically upon semiconductor material and fabrication process. Thus, it

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Nomenclature

ACE_{cal}	absolute current error of calculated current (A)
ACE_{sim}	absolute current error of simulated current (A)
c	centroid of simplex
D	problem dimension
f_M	error function of model
F	objective function value
F_0	objective function value at start point
I	terminal current (A)
I_0	diode saturation currents (A)
I_{cal}	calculated current (A)
I_D	diode currents (A)
I_{ph}	photocurrent (A)
I_{sim}	simulated current (A)
k	Boltzmann constant ($1.3806503 \times 10^{-23}$ J/K)
LB	lower bound
LBSDM	Lambert W function based SDM
m	factorial series of STF
MaxNFEs	maximum number of function evaluations
MNMS	modified Nelder-Mead simplex
MPP	maximum power point
n	diode ideality factor
N	number of measured I–V data
N_s	number of cells in series
N_p	number of uniform sampling points
N_r	allowed number of iterations
$NFEs$	number of function evaluations
NFE_{th}	number of function evaluations at thV
NMS	Nelder-Mead simplex
P	population of N_p sample points
q	electronic charge ($1.60217646 \times 10^{-19}$ C)
R_s	series resistance (Ω)
R_{sh}	shunt resistance (Ω)
$RMSEcal$	calculated root mean square error
$RMSEsim$	simulated root mean square error
S	bound-constrained simplex
S_e	bound-constrained expansion point
S_{ic}	bound-constrained inside contraction point
S_r	bound-constrained reflection point
S_{oc}	bound-constrained outside contraction point
S_u	unconstrained simplex
S_{ue}	unconstrained expansion point
S_{uic}	unconstrained inside contraction point
S_{uoc}	unconstrained outside contraction point
S_{ur}	unconstrained reflection point
$SBSDM$	Special Trans function based SDM
SDM	single diode model
STF	Special Trans function
thV	threshold value of $RMSE_{cal}$
$trans+$	symbol of Special Trans function
T	cell temperature (K)
UB	upper bound
V	terminal voltage (V)
V_{th}	thermal voltage (V)
wc	weighted centroid
W_0	principal branch of Lambert W function
x	STF branch
x_0	initial STF branch
X	unknown parameter vector
X_0	start point
X_{u0}	unconstrained surrogate of X_0
α	reflection coefficient
β	expansion coefficient
γ	contraction coefficient
δ	shrinkage coefficient

is essential to extract these parameters accurately from the measured I–V data of solar cells. Unfortunately, SDM Eq. (1) is an implicit transcendental equation and has the shortcoming of being explicitly unsolvable for either current I or voltage V only using common elementary functions [11]. This inherently implicit nature hampers not only solar cell parameter extraction [12] but also PV system simulation [2].

In order to circumvent this difficulty, some researchers have diverted their attention to the mathematical improvement of SDM Eq. (1). Authors in [13] reported a Lambert W function [14] based single diode model (LBSDM) Eq. (2).

$$I = \frac{R_{sh}(I_{ph} + I_0) - V}{R_s + R_{sh}} - \frac{nV_{th}}{R_s} W_0(U) \tag{2}$$

where W_0 is the principal branch of Lambert W function, and

$$U = \frac{I_0 R_s R_{sh}}{nV_{th}(R_s + R_{sh})} \exp\left[\frac{R_{sh}(R_s I_{ph} + R_s I_0 + V)}{nV_{th}(R_s + R_{sh})}\right] \tag{3}$$

Clearly, LBSDM Eq. (2) is an exact explicit expression of implicit

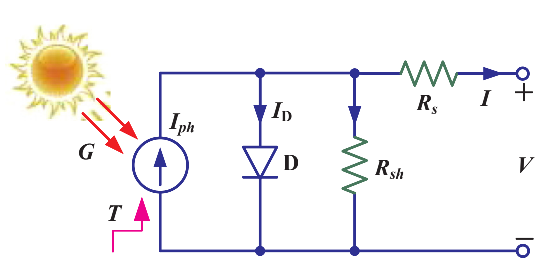

Fig. 1. The single diode model of a solar cell under illumination.

SDM Eq. (1). The application of LBSDM Eq. (2) have been summarized in one of our previous papers [15]. To sum up, LBSDM Eq. (2) has better accuracy and convergence than SDM Eq. (1). Regrettably, since the computational speed of Lambert W function is 2.8–4.1 times slower than that of exponential function [16], LBSDM Eq. (2) is computationally more expensive and time consuming than SDM Eq. (1) [17].

As an alternative to Lambert W function, the Special Trans function (STF) defined in [18] has been proved to be superior within Maple and Mathematica environment. This enables many transcendental equations involving exponential terms can be solved using STF. In particular, STF has been successfully applied to obtain the exact explicit expression of SDM Eq. (1). After some structural modifications, the STF based single diode model (SBSDM) reported in [19–23] can be rewritten as Eq. (4).

$$I = \frac{R_{sh}(I_{ph} + I_0) - V}{R_s + R_{sh}} - \frac{nV_{th}}{R_s} trans_+(x, U) \tag{4}$$

where $trans_+(x, U)$ is the STF defined as

$$trans_+(x, U) = U \frac{\sum_{m=0}^{[x]} U^m (x-m)^m / m!}{\sum_{m=0}^{[x+1]} U^m (x+1-m)^m / m!} \tag{5}$$

where $[x]$ denotes the greatest integer less than or equal to x. Since the number of accurate digits in the numerical structure of $trans_+(x, U)$ depends upon $[x]$ and for practical analysis [18], $trans_+(x, U)$ can be treated as a multi-branch function of $x \subset \mathbb{Z}^+$. Due to the factorial of m is only accurate for $m \le 21$ in commonly used double-precision arithmetic, in this paper the STF branch x is assigned to be a positive integer less than or equal to 20, i.e. $x \in \{1, 2, ..., 20\} \subset \mathbb{Z}^+$.

The common feature of LBSDM Eq. (2) and SBSDM Eq. (4) is that for any value of voltage V, the corresponding exact value of current I can be