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# Performance improvement of a four-terminal thermal amplifier with multiple energy selective tunnels

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ABSTRACT

A new model of the energy selective electron (ESE) device composed of four electron reservoirs and four energy filters is proposed. The device can work as a thermal amplifier, which may be equivalent to a coupling system consisting of two three-electron-reservoir heat pumps. Expressions for the heat-pumping rate (HPR) and coefficient of performance (COP) of the device are analytically derived through the heat flows of two-terminal ESE devices. The maximum HPR and COP are calculated. It is found that the maximum HPR of the coupling system can attain 2 times of that of a three-electron-reservoir heat pump while its COP is still equal to that of a three-electron-reservoir heat pump. Moreover, the effects of the chemical potentials of electron reservoirs, central levels of energy filters, half width at half maximum of energy filters on the HPR and COP of the device are discussed in detail. The optimal configuration of the device is determined. The cut-off value of the half width at half maximum of energy filters is calculated. The optimum selection criteria of main parameters are provided.

## 1. Introduction

Thermoelectric devices [1–4] may be used as both refrigerators and power generators [5–7]. The problem how to enhance the conversion efficiency of thermoelectric devices has attracted great attention. With the development of nanotechnology in recent years, many new appearing materials [8–10] with nanostructure can effectively improve the conversion efficiency of thermoelectric devices [11,12]. On the other hand, novel concepts [13–16] were constantly used in the design and fabrication of thermoelectric devices. For example, energy selective electron (ESE) devices [17–19] are a class of the thermoelectric devices with innovative design. By using an appropriate energy filter, it was predicted that ESE engines can be reversibly operated with the Carnot efficiency [17–19]. A series of experimental [20–23] and theoretical [24–26] works related to ESE devices were carried out.

At present the ESE devices [27–29] consisting of two heat reservoirs with two different chemical potentials connected by an energy filter is a class of the most studied ESE devices. It was found that the performance of a two-terminal ESE engine can be greatly improved by adding an electron reservoir and an energy selective tunnel [13]. When such a design concept was used in ESE refrigerators [30,31], the performance of refrigerators can be also improved effectively. It will be found that such a design concept is very significant for an ESE thermal amplifier [32].

In this paper, a new model of the thermal amplifier consisting of four electron reservoirs connected by four energy selective tunnels is proposed. It is expounded that such a thermal amplifier may be equivalent to a coupling system consisting of two three-electron-reservoir heat pumps. It is proved that the heat-pumping rate (HPR) of the coupling system can be doubled while the coefficient of performance (COP) of the coupling system is still maintained to be equal to that of a three-electron-reservoir thermal amplifier. Such a property of the thermal amplifier is similar to that of the electronic cooling device proposed in Ref. [33], in which the cooling power is doubled without reducing the COP of the cooling device. Moreover, the effects of some parameters on the HPR and COP of the amplifier are discussed in detail. The maximum HPR and COP of the thermal amplifier are determined. The selective criteria of main parameters, which are helpful to the design and operation of the thermal amplifier, are provided.

## 2. Model description

The ESE device considered here is composed of four electron reservoirs marked by $H$, $C$, $L$, and $R$ and four ESE tunnels labeled as $\varepsilon_{HL}$, $\varepsilon_{HR}$, $\varepsilon_{LC}$, and $\varepsilon_{RC}$, as shown in Fig. 1(a), where the temperatures and chemical potentials of electron reservoirs $H$ and $C$ are, respectively, ($T_H$, $\mu_H$) and ($T_C$, $\mu_C \equiv \mu_0$), electron reservoirs $L$ and $R$ have the same temperature $T_P$ but different chemical potentials $\mu_L$ and $\mu_R$, and the temperatures of electron reservoirs satisfy a relation: $T_H > T_P > T_C$. Electron reservoirs can exchange electrons through energy filters. The black

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## Nomenclature

| | |
|---|---|
| $f_i$ | Fermi-Dirac function |
| $h$ | Planck constant, J s$^{-1}$ |
| $k_B$ | Boltzmann constant, J K$^{-1}$ |
| $q_P$ | HPR, J s$^{-1}$ |
| $q_{P,m}$ | HPR at maximum COP, J s$^{-1}$ |
| $q_{P,\max}$ | maximum HPR, J s$^{-1}$ |
| $\dot{S}$ | overall entropy production rate, J K$^{-1}$s$^{-1}$ |
| $T_i$ | temperature, K |

*Greek symbols*

| | |
|---|---|
| $\delta$ | half width at high maximum of energy filter, eV |
| $\delta_C$ | cut-off value of $\delta$, eV |
| $\varepsilon$ | centre energy level of energy filter, eV |
| $\varepsilon_{ij}$ | resonant level between the reservoirs $i$ and $j$, eV |
| $\gamma_{ij}$ | transmission function of energy filter |
| $\mu_i$ | chemical potential, eV |
| $\mu_0$ | chemical potential of electron reservoirs C, eV |
| $\psi$ | COP of amplifier |
| $\psi_{\max}$ | maximum COP |

*Subscripts*

| | |
|---|---|
| C | reservoir C |
| H | reservoir H |
| $i$ | H, R, L, C |
| $j$ | $j \neq i$, H, R, L, C |
| L | reservoir L |
| R | reservoir R |

*Abbreviations*

| | |
|---|---|
| COP | coefficient of performance |
| ESE | energy selective electron |
| HPR | heat-pumping rate |

arrows represent the directions of electron flows, and the empty white arrows represent the directions of energy flows. When $\varepsilon_{ij} > \mu_i$ and $\varepsilon_{ij} > \mu_j$ $(i,j = H,C,L,R,\ i \neq j)$, the direction of the electron flow is the same as that of the energy flow and the electron flow is indicated by red balls. When $\varepsilon_{ij} < \mu_i$ and $\varepsilon_{ij} < \mu_j$, the direction of the electron flow is opposite to that of the energy flow and the electron flow is indicated by blue balls.

For the ESE device with a single energy filter, when the temperature $T_i$ of the electron reservoir $i$ at chemical potential $\mu_i$ is higher than that $T_j$ of the electron reservoir $j$ at chemical potential $\mu_j$, the heat flows from the electronic reservoirs $i$ and $j$ to the ESE device can be, respectively, expressed as

$$q_{ij} = \frac{2}{h}\int_{-\infty}^{\infty}(f_i - f_j)(\varepsilon - \mu_i)\gamma_{ij}d\varepsilon, \tag{1}$$

and

$$q_{ji} = \frac{2}{h}\int_{-\infty}^{\infty}(f_j - f_i)(\varepsilon - \mu_j)\gamma_{ij}d\varepsilon, \tag{2}$$

where

$$f_i(\varepsilon,\mu_i,T_i) = \frac{1}{\exp[(\varepsilon - \mu_i)/(k_B T_i)] + 1} \tag{3}$$

is the Fermi-Dirac function of the electron reservoir [31], and

$$\gamma_{ij} = \frac{1}{1 + (\varepsilon - \varepsilon_{ij})^2/\delta^2} \tag{4}$$

is the transmission function of the energy filter, $\varepsilon_{ij}$ are the resonant level between the reservoirs $i$ and $j$, and $\delta$ is the half width at half maximum of the resonant level [34].

According to Fig. 1(a) and the symbolic rules of $q_{ij}$ and $q_{ji}$, the thermal amplifier shown in Fig. 1(a) may be equivalent to a coupling system consisting of two three-electron-reservoir heat pumps, as shown in Fig. 1(b), where the first three-electron-reservoir heat pump is composed of a heat engine operating at two electron reservoirs at temperatures $T_H$ and $T_P$ and chemical potentials $\mu_H$ and $\mu_L$ and a heat pump operating at two electron reservoirs at temperatures $T_P$ and $T_C$ and chemical potentials $\mu_L$ and $\mu_0$, and the second three-electron-reservoir heat pump is composed of a heat engine operating at two electron reservoirs at temperatures $T_H$ and $T_P$ and chemical potentials $\mu_H$ and $\mu_R$ and a heat pump operating at two electron reservoirs at temperatures $T_P$ and $T_C$ and chemical potentials $\mu_R$ and $\mu_0$. The net heat flows from the electron reservoirs $H$ and $C$ are $q_H = q_{HL} + q_{HR}$ and $q_C = q_{CL} + q_{CR}$, and the net heat flows entering into the electron reservoirs $L$ and $R$ are $q_L = -q_{LH} - q_{LC}$ and $q_R = -q_{RH} - q_{RC}$. Thus, the HPR and COP of the thermal amplifier are, respectively, expressed as

$$q_P = q_L + q_R = -q_{LH} - q_{LC} - q_{RH} - q_{RC} \tag{5}$$

and

$$\psi = \frac{q_P}{q_H} = -\frac{q_{LH} + q_{LC} + q_{RH} + q_{RC}}{q_{HL} + q_{HR}} = \alpha\psi_L + (1-\alpha)\psi_R \tag{6}$$

where $\alpha = q_{HL}/q_H$, $\psi_L = q_L/q_{HL}$ is the COP of the first three-electron-reservoir heat pump, and $\psi_R = q_R/q_{HR}$ is the COP of the second three-electron-reservoir heat pump.

According to Fig. 1, the entropy production rate of the thermal amplifier is given by

$$\dot{S} = \frac{q_P}{T_P} - \frac{q_H}{T_H} - \frac{q_C}{T_C} \tag{7}$$

and the following equations

$$\int_{-\infty}^{\infty}(f_H - f_R)\gamma_{HR}d\varepsilon = \int_{-\infty}^{\infty}(f_R - f_C)\gamma_{RC}d\varepsilon = \int_{-\infty}^{\infty}(f_C - f_L)\gamma_{LC}d\varepsilon = \int_{-\infty}^{\infty}(f_L - f_H)\gamma_{HL}d\varepsilon \tag{8}$$

can be directly obtained by the conversation of electron numbers.

## 3. Results and discussion

When $\delta$ is very small, Eqs. (5)(8) can be, respectively, simplified as

$$q_P = \frac{2\pi\delta}{h}(f_H - f_R)(\Delta\varepsilon_H + \Delta\varepsilon_C) = \frac{4\pi\delta}{h}(f_H - f_R)(\Delta_H - \Delta_C - \varepsilon_{HL} + \varepsilon_{LC}) = \frac{4\pi\delta}{h}(f_H - f_R)(-\Delta_H + \Delta_C + \varepsilon_{HR} - \varepsilon_{RC}), \tag{9}$$

$$\psi = \frac{\Delta\varepsilon_H + \Delta\varepsilon_C}{\Delta\varepsilon_H} = \frac{\Delta_H - \Delta_C + \varepsilon_{LC} - \varepsilon_{HL}}{\Delta_H + \mu_0 - \varepsilon_{HL}} = \frac{\Delta_H - \Delta_C - \varepsilon_{HR} + \varepsilon_{RC}}{\Delta_H + \mu_0 - \varepsilon_{HR}}, \tag{10}$$