Energy Conversion and Management 164 (2018) 570–578

Contents lists available at ScienceDirect

Energy Conversion and Management

journal homepage: www.elsevier.com/locate/enconman

A new estimation approach based on moments for estimating Weibull parameters in wind power applications

Ilhan Usta[a], Ibrahim Arik[b], Ismail Yenilmez[a], Yeliz Mert Kantar[a,*]

[a] Anadolu University, Science Faculty, Dep. of Statistics, 26470 Eskisehir, Turkey
[b] Bilecik Seyh Edebali University, Science and Art Faculty, Dep. of Statistics, 11230 Bilecik, Turkey

ARTICLE INFO

Keywords:
Weibull distribution
Moments
Multi-objective function
Wind power
Comparative analysis

ABSTRACT

This study introduces a new estimation approach that could be used in calculating the Weibull parameters for the estimation of wind power. This new approach, called the method of multi-objective moments (MUOM), proposes minimization of the squared deviances between the first three population moments and corresponding sample moments. In the proposed MUOM, the third moment, which is used in the formula of wind power, is a key function since it provides a significant contribution to the correct prediction of wind energy. In order to demonstrate the performance of the proposed MUOM, it is compared with the well-known estimation methods such as the maximum likelihood, modified maximum likelihood, moment and energy pattern factor. In comparative analysis, the performance of the considered methods is evaluated on the actual wind speed data measured different time periods according to various goodness of fit criteria. The obtained results indicate that the MUOM definitely provides more accurate estimates than other well-known methods in estimating wind power based on the Weibull distribution. Thus, the MUOM can be used as an improved method in estimating wind power.

1. Introduction

The Weibull distribution (WD) has been commonly-used to characterize different types of data such as wind speed data, reliability data, survival data and ceramics, glasses and solid catalysts data [1–3]. Particularly, the WD is widely-used distribution in the estimation of wind energy potential in a specified region, considering its flexibility and easy computation, as well as it being a good fit to wind data [4]. Thus, the parameter estimation methods of the WD have been intensely studied in the literature [2–19]. While some studies have proposed new estimation method for the parameters of the WD, most of them have compared existing methods in terms of different angles. For instance, Seguro and Lambert [5] have compared the well-known maximum likelihood method (MLM), graphical method (GM) and the proposed modified maximum likelihood method (MMLM). They found that the MLM provides better performance than the others. Tizgui et al. [6] have investigated the performances of the GM, MLM, the empirical method of Justus (EMJ), the empirical method of Lysen (EML), the energy pattern factor method (EPFM), Mabchour's method and the method of moments (MM). Same methods have also been evaluated in Kaplan [7]. Chalamcharla and Doraiswamy [8] have derived probability distribution for the power output with the WD estimated by MLM. On the other hand, Mohammadi et al. [9] have evaluated the effectiveness of six numerical methods for calculating the wind power density. They found that EMJ, EML, EPFM and MLM present very favorable efficiency in estimating wind power. Saleh et al. [10] have deduced that the mean wind speed method and the MLM are recommended in estimating the wind speed distribution. In addition, Arslan et al. [11] have assessed the performances of the MM, MLM and L-moment method for estimation of the parameters of the WD. They reached that the MLM for large sample sizes is preferable in terms of the mean square error criterion when the MLM is compared with the other methods for the estimation of shape parameter, as in [12]. Furthermore, GM, MLM, MMLM, MM and EMJ have been compared in Chang [13] for estimating the parameters of the WD for wind energy application. Similarly, Costa Rocha et al. [14] have studied the equivalent energy method with the same six methods evaluated by Chang [13]. They aimed to analyse and compare these seven numerical methods on wind speed data observed in the northeast region of Brazil. Besides, while Azad et al. [15] have reached that the MM and MLM are the most efficient methods for the WD, De Andrade et al. [16] have studied some methods considered in [14,15] for the WD, using wind speed data measured in the two coastal cities of northeast region of Brazil. On the other hand, Werapun et al. [17] again have compared some existing estimation methods for the WD, they

* Corresponding author.
E-mail address: ymert@anadolu.edu.tr (Y.M. Kantar).

https://doi.org/10.1016/j.enconman.2018.03.033
Received 15 January 2018; Received in revised form 1 March 2018; Accepted 13 March 2018
0196-8904/

Nomenclature

v	wind speed (m/s)
V	random variable
v_i	*ith* observed wind speed data
n	number of all observed non-zero wind speed data
N	number of bins
Pdf	probability density function
Cdf	cumulative distribution function
WD	Weibull distribution
c	scale parameter of WD
k	shape parameter of WD
$f_{WD}(v)$	Weibull probability density function (pdf)
$F_{WD}(v)$	Weibull cumulative distribution function (cdf)
$\Gamma(.)$	gamma function
A	wind turbine blade sweep area (m^2)
ρ	air density (kg/m^3)
P_{REF}	mean power density based on time series wind data (Ws/m)
P_D	mean power density based on WD (Ws/m)
x_j	jth observed probability $j = 1, \dots, n$
y_j	jth predicted probability calculated from a special distribution
z_j	jth computed value from the correlation equation for the same value of x_j
R^2	coefficient of determination
CHI	chi-square test
RMSE	root mean square error
KS	Kolmogorov-Smirnov distance
PDE	power density error
MLM	maximum likelihood method
GM	graphical method
MMLM	modified maximum likelihood method
MM	method of moments
EPFM	energy pattern factor method
MUOM	method of multi-objective moments

reached that the GM is showing the worst performance among these methods. As different from other studies, while Akdag and Dinler [19] have developed a new method, which provides less power density error than the other considered methods, based on the wind power formula of the WD, Usta [18] has proposed a new method, which considers probability weighted moments with the power density method, and compared the proposed method with existing ones.

When reviewed the literature of the WD in energy studies, it can be seen that there are many studies which focus on the parameters of the WD to estimate wind power accurately, however, there is no generally agreed upon consensus on what the best estimation method for the WD is.

In this study, a new estimation approach for the WD parameters in estimating wind power is introduced. This new estimation approach, called the method of multi-objective moments (MUOM), is developed with the idea of minimization of the squared deviations between the first three population moments and corresponding sample moments. In the proposed MUOM, the third moment, which is used in the formula of wind power, is a primary function since it aims providing accurate estimation of wind power.

Considering all these issues, the study is organized as follows: Section 2 reviews the previous estimation methods: the maximum likelihood, modified maximum likelihood, moment and energy pattern factor and also introduces the new estimation approach, MUOM for the WD. Model selection criteria and statistics, which are the coefficient of determination (R^2), root mean square error (RMSE), Kolmogorov-Smirnov (KS) and Chi-square (CHI), and power density error (PDE) are provided in Section 3. The performance of the MUOM and other well-known methods is evaluated in Section 4 on the actual wind speed data measured in Turkey at different period. Section 5 concludes the study with the obtained results.

Table 1
The formulas of criteria for model evaluation.

Criteria	Formulas
R^2	$1-\sum_{i=1}^{N}(y_i-x_i)^2/\sum_{i=1}^{N}(y_i-z_i)^2$
KS	$\max_{1\leqslant i\leqslant n}(F(v_i)-(i-1)/n, i/n-F(v_i))$
RMSE	$\left(\sum_{i=1}^{N}(y_i-x_i)^2/N\right)^{1/2}$
CHI	$\frac{\sum_{j=1}^{N}(y_j-x_j)}{N}$
PDE	$\left\lvert\frac{P_{REF}-P_D}{P_{REF}}\right\rvert \times 100,$

Fig. 1. Locations of wind speed observations used in this study.

2. Methods for estimating Weibull distribution parameters

The WD has been commonly used and accepted-reference distribution in the field of wind energy [20]. The probability density function (pdf) and cumulative distribution function (cdf) for the Weibull random variable, are given in Eqs. (1) and (2), respectively:

$$f_{WD}(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^k}, \quad v,c,k > 0 \tag{1}$$

$$F_{WD}(v) = 1-e^{-\left(\frac{v}{c}\right)^k} \tag{2}$$

where V is the Weibull random variable, k and c are the shape and scale parameters of the WD, respectively.

Also, the *rth* moment of the WD is given as follows:

$$E(V^r) = c^r\Gamma\left(1+\frac{r}{k}\right) \tag{3}$$

Since wind power estimation is formulated as an explicit function of the estimated parameters of the WD, accurate estimation of its parameters is important in order to obtain correct results regarding wind energy potential.