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Performance analysis of wind turbines at low tip-speed ratio using the Betz-Goldstein model

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ABSTRACT

Analyzing wind turbine performance at low tip-speed ratio is challenging due to the relatively high level of swirl in the wake. This work presents a new approach to wind turbine analysis including swirl for any tip-speed ratio. The methodology uses the induced velocity field from vortex theory in the general momentum theory, in the form of the turbine thrust and torque equations. Using the constant bound circulation model of Joukowsky, the swirl velocity becomes infinite on the wake centreline even at high tip-speed ratio. Rankine, Vatistas and Delery vortices were used to regularize the Joukowsky model near the centreline. The new formulation prevents the power coefficient from exceeding the Betz-Joukowsky limit. An alternative calculation, based on the varying circulation for Betz-Goldstein optimized rotors is shown to have the best general behavior. Prandtl's approximation for the tip loss and a recent alternative were employed to account for the effects of a finite number of blades. The Betz-Goldstein model appears to be the only one resistant to vortex breakdown immediately behind the rotor for an infinite number of blades. Furthermore, the dependence of the induced velocity on radius in the Betz-Goldstein model allows the power coefficient to remain below Betz-Joukowsky limit which does not occur for the Joukowsky model at low tip-speed ratio.

1. Introduction

It is well-known that mathematical models for horizontal-axis wind turbines (HAWTs) design have increased in the last century [1,2]. Most models are based on the general momentum theory with some simplifying hypothesis for the complex flow in the wake [3,4]. As the tip-speed ratio $X = \Omega R/V_0$ decreases, the relative magnitude of the swirl velocity increases, which makes this region of turbine operation particularly challenging to analyze. In the definition of X, Ω and R are the angular velocity and tip radius of the blades, respectively, and V_0 is the wind speed.

There have been a number of analyses of HAWT operating at low X, and least two, Wood [5] and Rio Vaz et al. [6], have suggested that the power coefficient may exceed the Betz-Joukowsky limit of 59.3% as $X \downarrow 0$. This behavior is associated with the modeling of the trailing vorticity. The swirl, or circumferential velocity, represents the angular momentum generated by the blades and hence is directly related to the torque and power extracted by the rotor. The commonly used "free-vortex" assumption for the wake is equivalent to the Joukowsky model of the blades in which the bound circulation is constant. This causes an infinite velocity along the axis of rotation. Glauert [7] avoided this problem by terminating the integration of the blade element equations for momentum and angular momentum at the radius where the angular velocity of the swirl, ω, equals Ω. Wilson and Lissaman [8] split the wake using Ω/ω_{max}. As for a Rankine vortex, the swirl velocity near the axis is now proportional to the radius, r, and no infinities occur. The strong influence of swirl on the angular momentum balance for a rotor carries over to the energy and thrust equations.

The power coefficient must be zero at zero X and it is considered as axiomatic that the power coefficient, C_P asymptotes to the Betz-Joukowsky limit as X becomes large. All references to maximizing performance in this paper will be to maximizing C_P. Glauert [7] determined its monotonic increase with X to approach the Betz-Joukowsky limit for an ideal rotor with an infinite number of blades, B. The simple analysis that leads to the Betz-Joukowsky limit ignores swirl. This can be justified on the grounds that HAWTs at high X produce power by the multiplication of a small torque and a high angular velocity, but the accuracy of this approximation has never been made clear. Wood [5] proposed a more general model for the hub vortex, and suggests that some account of the vortex structure of the wake will be required to resolve fully

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Nomenclature

a, a'	axial and tangential induction factors at the disc
a_b, a'_b	average axial and tangential induction factors at the rotor
A, A_1	area of the disc and the cross section far in the wake
b	axial induction factor in the wake
B	number of blades
C_P	power coefficient
C_Q	torque coefficient
C_T	thrust coefficient
dC_P	local power coefficient
dC_Q	local torque coefficient
dC_T	local thrust coefficient
dQ	local torque (N m)
dT	local thrust (N)
dr	radial increment at the rotor (m)
dr_1	radial increment in the far wake (m)
F	tip-loss factor
F_P	Prandtl tip-loss factor
p	dimensionless pitch for the helical vortex
p_0	pressure in the free stream (Pa)
p_1	pressure in the wake (Pa)
q	dimensionless circulation
r	radial position at the disc (m)
r_1	radial position in the wake (m)
r_c	vortex core radius (m)
S	swirl number
u	axial velocity at the disc (m/s)
u_1	axial velocity in the far wake (m/s)
u_θ	swirl velocity in the near wake (m/s)
$u_{\theta 1}$	swirl velocity in the far wake (m/s)
$u_{\theta,max}$	maximum swirl velocity in the wake (m/s)
V_0	freestream velocity (m/s)
X	tip-speed ratio
$\overline{w}$	w/V_0, normalized velocity of the vortex sheet
Greek Symbols	
α	exponent in Vatistas vortex model
ξ	r/r_c
ρ	density of air (kg/m^3)
ϕ	flow angle (rad)
ω	angular speed of the flow in the wake (rad/s)
Ω	angular speed of the disc (rad/s)
Γ	vortex circulation (m^2/s)

the effects of swirl. Based on the analytical solution for the velocities induced by helical vortex filament, Okulov and Sørensen [9] analyzed in detail the original formulation of the Goldstein function [10] for optimum rotor loading. They found that the model can be extended to heavily loaded rotors in full accordance with the general momentum theory at all X. Sørensen and Kuik [11] proposed a refined model based on the momentum including the friction and pressure on the lateral boundary of the annular streamtube intersecting each blade element. In their model, the lateral pressure is proportional to the area expansion and the pressure drop over the rotor at the edge of the disc, $\epsilon \Delta p|_{r=R}(A_1 - A)$. However, the term ϵ is not fully understood because it is not known a priori how to determine such a term, and the authors are unaware of reference with further details on ϵ. Rio Vaz et al. [6] present an optimization model taking into account the influence of the swirl, based in the formulation proposed by Wilson and Lissaman [8] for the power coefficient. They found that the efficiency of a HAWT is heavily influenced by wake rotation at small X. Mikkelsen et al. [12] used a CFD actuator line model to analyze the wake at low X. It was shown that the excessive swirl near the centreline at low X generates vortex breakdown, causing a recirculation zone in the wake that limits the power yield of the rotor. The appearance of vortex breakdown has a similar effect on the flow behavior as the vortex ring state that usually appears at higher X.

Wood's [13] lifting line analysis of optimal loading at very low X rediscovered Goldstein's [14] result for the asymptotic behavior of his function. Wood [13] found that the induced velocity is linear in radius for any number of blades. This inviscid analysis avoided the infinite velocities that usually result from the assumption of inviscid flow. Wood [15] considered the limiting case of the Goldstein and Glauert analyses as $X \downarrow 0$ for an ideal rotor with an infinite number of blades, B. He showed that the former is physically untenable as it requires significant energy extraction and a constant induced velocity for a stationary rotor. A modification of Goldstein's analysis produced a more satisfactory physical model in which the swirl was linear in r near the axis and inversely proportional to r at higher values of the local-speed ratio, X_r. The analysis, however, was limited to infinite B without wake expansion.

The analysis for finite B and wake expansion using an induced velocity field dependent on r are the main topics of this paper. A detailed analysis is developed for the swirl influence on the thrust, torque and power coefficients that is valid for all X. General analytical formulations are developed for the azimuthal velocity distributions of the Joukowsky [16] model combined with the Rankine, Vatistas [17] and Delery [18] models for the vortex core. The effect of finite B on the general theory is accounted for by using two formulations for the tip-loss factor: that of Prandtl [19] and the recent method of Wood et al. [20]. The results show that a swirl velocity dependent on r produces an acceptable physical behavior. The generalized formulations for the thrust, torque and power coefficients, are in good agreement with previous results. The torque coefficient has a maximum value of about 0.5 when B tends to infinity, which agrees with Wood [15].

The remainder of this paper is organized as follows. The next section introduces the general momentum theory, showing the expressions for the performance coefficients using the Joukowsky model with Rankine, Vatistas and Delery vortices. Section 2 describes the general form of the Betz-Goldstein (BG) model. Section 3 shows the results and discussion, where the dependence on radius of the induced velocity field is presented, as well as the integral form of the performance coefficients taking into account the tip-loss factor. Section 4 shows the conclusions of this study.

2. General formulations for the performance coefficients

General momentum theory differs from the simple axial momentum theory that leads to the Betz-Joukowsky limit, by accounting for the rotational motion. To achieve this, it is necessary modify the usual actuator disc theory. A flow model that includes the angular momentum, was presented in Joukowsky [16], and applied by Glauert [7] for the study of propellers, and later modified by Wilson and Lissaman [8] for HAWTs. In that case, the induction factor in the far-wake has a non-linear relationship