On the Definition of Curve's Frame at the Points of inflection

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Abstract:
This paper deals with the problem of defining a frame along the curve at the point of inflection such that the intrinsic properties of the curve are reflected in the defined frame and the frame can be used as a local frame of the body moving on the curve. It is natural to use the Frenet Serret Frames of the curve whenever we need to define a local reference frame along the curve, but the problem with Frenet Serret Frame is that, it is not defined at the point of inflection. The problem gets serious if the point of inflection drags along for some times. i.e. if the direction of the unit tangent is not changing for some time. In such cases if we want to use the Frenet frame of the curve as the local reference frame of a body (a local frame for some sensor e.g camera reference frame) moving along the same curve, it will suffer from the problem of indeterminacy at the point of inflection. Therefore we have to define a frame which exists at all the points of the curve. This defined frame must retain the intrinsic properties of the curve. The Rotation Minimizing Frame is proposed as a solution to the problem in this paper.

KEYWORDS: Pythagorean Hodograph Curves, Point of inflection, Frenet Frame, Rotation Minimizing Frame, Rotation Matrix, Intrinsic properties of curve.

1. INTRODUCTION:

The process of path planning for UAVs results in curves with certain properties (tangential and curvature continuity) as described by M.shah at el [2009], S. Madhavan at el [2006] and A. Armando at el [2009]. The path planning does not demand the curve to fulfill the requirement that at each of its point a frame must be defined which can be used as a local reference frame, but if a sensor is on board the vehicle there is a need for the frame to be defined at each point of the curve which can act as local reference frame for the sensor such that all the calculations made in sensor local reference frame can be transformed to universal reference frame using rotation matrix. For example if a camera is used as a sensor then a frame is needed to be defined in which all the output of the camera is referenced Amanuele Trucco[1998].

Generally the Frenet Serret frame is used as a local frame for the sensor as described by Z. Duric at el [1998]. The Frenet Serret frame at each point of a regular curve *r(t)* is defined as an orthonormal basis $(\vec t, \vec n, \vec b)$ in R^3 aligned with local intrinsic curve geometry as described by R. T. Farouki at el [2008]. The elements of this basis are the curve unit tangent vector $\vec t$ unit normal vector $\vec n$ and a unit binormal vector $\vec b$ given by:

$$\vec t = \frac{r'}{|r'|} \quad (1)$$

$$\vec n = \frac{r' \times r''}{|r' \times r''|} \quad (2)$$

$$\vec b = \vec t \times \vec n \quad (3)$$

On the regular *(r(t) ≠ 0 for all t)* the unit tangent vector is defined at every point ,but the unit normal vector and unit binormal vector are not defined at the point of inflection.

The point of inflection is the point on the curve at which the unit normal reverses its direction or at which the direction of unit tangent is th same as the direction of unit normal vector.

At the point of inflection r'*(t)* becomes parallel to *r''(t)*. In such case the cross product:

$$r' \times r'' = 0$$

in the expression for unit normal vector (equation 2). Consequently the unit binormal vector becomes undefined as well. This will make the Frenet frame to be undefined. In fact $\vec n$ and $\vec b$ experience a sudden reversal upon passing through point of inflection.

In this paper the Frenet frame is consider to be the body local frame, with origin at canter of mass of the body. If we assume to place the camera at the center of mass of the UAV (for simplicity) then the Frenet frame is also acting as camera reference frame in which all the measurements of

the perception system are made and then transformed to universal frame using rotation matrix John Craig [1989]. For a curve having no inflection point, this is a perfect idea. However for a curve which has a point of inflection this create problem because the Frenet frame is not defined at the inflection point. Consequently the camera frame becomes undefined and the outputs of the perception system become meaningless. This problem is further aggravated if the tangent vector $\vec{t}$ and the normal vector $\vec{n}$ remain parallel for some time or in other words if the point of inflection drag along the curve for some time.
The problem cam be solved by defining a frame which is defined at every point of the path. In this paper Rotation Minimizing Frame [2003] is used to overcome this problem.
The rest of the paper is organized as follows: Section 2 describes the problem formulation. Section 3 introduces the Pythagorean Hodograph. Section 4 introduces Rotation minimizing Frames as solution to the problem of indeterminacy of Frenet frame at the point of inflection. Section 5 is about generating the Rotation Minimizing Frame for Pythagorean Hodograph quintic curves. Section 6 presents simulation results obtained with the proposed techniques. Section 7 comments on conclusion.

2. PROBLEM FORMULATION (SCENARIO)

A mission is planned to fly a group of unmanned aerial vehicles safely from base B with initial pose $P_{si}(x_{si}, y_{si}, z_{si}, \theta_{si}, \phi_{si})$ to target T with final pose $P_{fi}(x_{fi}, y_{fi}, z_{fi}, \theta_{fi}, \phi_{fi})$ as shown in Figure 1.

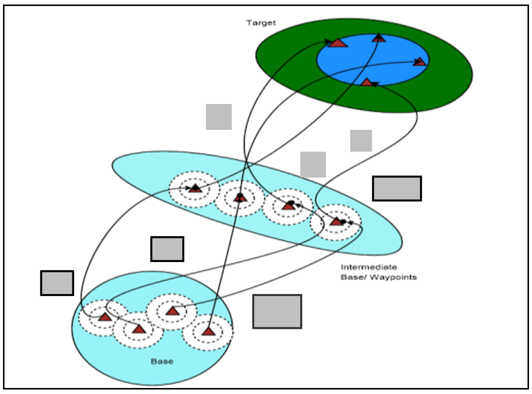

Figure 1: Swarm of UAVs Scenario

All vehicles start from the base at the same time. The flight of the UAV is in the environment which contains stationary and moving obstacles with a partial or no knowledge about their locations. During the flight, the vehicles will avoid inter-collision and collision with the stationary and moving obstacles. The absence of the complete knowledge about the environment necessitates the perception. Therefore each vehicle is provided with perception system comprising of a camera and a fusion algorithm to get, for example, vision information and motion estimation of unknown obstacles (pop up threats).
The UAVs paths are planned offline by the path planning module (Path Planner) using differential geometry technique of Pythagorean Hodograph on the basis of the available knowledge about their poses and the environment.
The UAVs start following these paths, if during the flight any of the UAVs comes across an obstacle, which was not known before then the camera, is used to locate these obstacles so that changes could be made to the initial path to avoid the obstacles.
This is where a local reference frame is needed to be defined for the camera. The PH path which does not have any point of inflection there is no problem, but the PH path which contain an inflection point the perception system suffer from the problem of indeterminacy of local reference frame because the Frenet Frame is not defined at such point.
It should be bear in mind that with the path planning perspective alone, the inflection point poses no problem, because the inflection point only demand a change of direction of flight of UAV. The UAV can fulfill this demand inside the limits of its dynamic constraints. However the perception system needs a defined frame at every point of the curve to which all of its outcome could be referenced.

3. PYTHAGOREAN HODOGRAPHS

Since Pythagorean Hodographs (PH) [1990, 1994] are used to calculate the paths of UAVs and the Rotation Minimizing Frame are calculated for these PH paths therefore a brief introduction of Pythagorean Hodograph is given in this paper to put the reader in the context.
Pythagorean Hodographs are the first derivatives of parametric polynomials, which satisfy the Pythagorean condition. If r(t) represents the curve and s(t) represents the length of the curve from instant t_1 to instant t_2 then:

$$s(t) = \int_{t2}^{t1} | r'(t) | \, dt$$

$$= \int_{t1}^{t2} \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2} \, dt \qquad (4)$$

x(t), y(t) and z(t) are the parametric components of the curve r(t), t is the parameter of the curve and x', y' an z' are the hodographs. Since the hodograph are Pythagorean therefore radical sign can be eliminated from equation (4):

$$\sigma^2(t) = x'^2(t) + y'^2(t) + z'^2(t) \qquad (5)$$

σ is the parametric speed.
This is Pythagorean law, if the polynomial σ is taken as the hypotenuse and the hodographs are