Solution of Operation, Planning and Control Problems of Large Power Systems in Distributed Control System

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Abstract: In this paper the functional modeling (FM) method developed for efficient solution of simulation, planning and control problems in large power systems and interconnections is presented. According to the FM method a power system is represented by hierarchical model, consisting of systems of equations of subsystems (lower level) and system of equations of higher level, in which only boundary variables of subsystems are present. Combined with Subgramian calculation this approach forms the basis for hierarchical distributed control system.

Keywords: stability analysis, power systems, Lyapunov equation, Gramians, distributed processing, economic dispatch

1. INTRODUCTION

For a long time the growth of size and complexity of power systems followed by increasing number of generating and transmission objects and relations between them was accompanied by improving of centralized control systems.

Developments in information technology have enabled dispatchers to gather information about all the parameters of the energy system, to accumulate and analyze multiple factors in the management of the grid.

Now you can see the parameters of the power system in real time resulted in the introduction of automatic control systems of the planned power stations, as well as automatic frequency control and balance, power flows.

All this information is collected on the upper level control, allowing you to deal with more accurate and detailed models of power systems. But in the way of increasing the flow of information are obstacles that seem to have to force us to abandon further concentration of data in a single control center.

The energy flows were oriented in the same direction so far - from large stations through the network to consumers. Management systems have changed only the intensity of energy flows according to changing consumption or emergency situations. The development of distributed generation makes the network bi-directional and in the recent situation centralized control seems to be deterrent to the effectiveness of the power system.

Classical approach to solution of stability analysis problems in power system is based on analysis of properties of a system of equations, representing dynamic behavior of a power system as a whole (Kundur (1994), Misrihanov et al. (2008), Lyapunov (1934)). This general approach being adequate and efficient in application to dynamic analysis of small and medium-size power systems meets with difficulties when applied to solution of these problems in large power interconnections.

The problems solutions properties analysis leads us to the power systems Gramians investigation. This paper is being considered the problem of developing the tool for large power systems control using Subgramian calculation combined with functional modeling approach.

2. SUBGRAMIAN METHOD IMPLEMENTATION FOR DISTRIBUTED CONTROL SYSTEM

Classical Lyapunov approach allows to bring together static stability problem to the solution of the Lyapunov matrix equation (Lyaponuv, A. (1934)). As for Lyapunov matrix equation, the problem can be solved by Bartels et al. (1972), or Golub et al. (1979) methods. The problems solutions properties analysis leads to power systems gramians exploration.

Let us suppose, that mathematical model of the power system is defined by the nonlinear algebraic differential equations system

$$\dot{x} = f(x, u, t), \qquad x(t_0) = 0,$$
$$M(x, t)x(t) = N(x, t)u(t) \tag{1}$$

Linearized model in small relative to the power system fixed mode is defined as the linear algebraic-differential equations system

$$\dot{x} = Ax(t) + Bu(t), \qquad x(t_0) = 0,$$
$$Mx(t) = Nu(t) \tag{2}$$

Suppose that, matrix N is nonsingular one. Then equations system (2) may be transformed as

$$\dot{x} = A_1 x \tag{3}$$

$$A_1 = A + BN^{-1}M \tag{4}$$

Let us suppose also, that system (2) is fully controllable and observable, and consider an autonomous system

The decay rate or the degree of stability of the system (3) is defined to be the largest d, such that the limit expression

$$\lim_{t\to\infty} e^{dt} \|x(t)\| = 0, \tag{5}$$

holds for all trajectories $x(t)$.

$$A_1^{\mathrm{T}} P + PA_1 + 2dP \le 0, \tag{6}$$

The first way to solve the problem (6) is based on the optimization problem solution with constrains as follows (Boyd et al. (1994))

$$\underset{subject\ to\ P\ge 0}{\text{maximize d}} \tag{7}$$

The second way is based on the following theorem (Andreev (1976))

Theorem 1. The matrix A_1 has the degree of stability d if and only if a positive-definite matrix solution V of Lyapunov algebraic equation exists for arbitrary positive-definite matrix W.

$$(A_1 - dI)^{\mathrm{T}} V + V(A_1 - dI) = -W \tag{8}$$

If one choose matrix W as unit matrix I, equation (8) can be written as

$$(A_1 - dI)^{\mathrm{T}} V + V(A_1 - dI) = -I \tag{9}$$

Given matrix $A_1 - dI$ is Hurwitz one, the equation (9) solution being Gramian for the system (3)

$$P = \int_0^{\infty} e^{(A_1 - dI)t} e^{(A_1 - dI)^{\mathrm{T}} t} dt \tag{10}$$

Let us consider other technique of equation (10) solution in time or frequency domain.

Lemma. (Yadykin (2010)). Let $\mathbf{A}_{[n\times n]}$, be real matrix and eigenvalues s_δ of $\mathbf{A}_{[n\times n]}$, be simple, the spectrum of the matrix A includes simple eigenvalues s_δ , the condition $s_k + s_\lambda \ne 0, \forall k = 1,2..n;\ \ \forall \lambda = 1,2...r$ are hold. Then at all finite times $\forall t \in [0, \infty)$ the following identities are true:

$$\int_0^T \mathbf{e}^{\mathbf{A}\boldsymbol{\tau}} \mathbf{e}^{\mathbf{A}_{\mathbf{m}}^{\mathrm{T}}\boldsymbol{\tau}} d\tau = \sum_{k=1}^{n} \sum_{j=0}^{n-1} \sum_{\eta=0}^{n_m-1} \frac{s_k^j (-s_k)^\eta}{N'(s_k) N_m(-s_k)} \mathbf{A}_{\mathbf{j}} \mathbf{A}_{\mathbf{m\eta}}^{\mathbf{T}} + \tag{11}$$

$$+ \sum_{k=1}^{n} \sum_{\lambda=1}^{n_m} \sum_{j=0}^{n-1} \sum_{\eta=0}^{n_m-1} \frac{s_k^j s_{m\lambda}^\eta}{(s_{m\lambda} + s_k) N'(s_k) N_m'(s_{m\lambda})} \mathbf{A}_{\mathbf{j}} \mathbf{A}_{\mathbf{m\eta}}^{\mathbf{T}} e^{(s_k + s_{m\lambda})T},$$

where $\mathbf{A}_j$ are Faddeev matrices, generated by expansion of resolvent of the matrix $\mathbf{A}$.

An expansion of matrix resolvents is known to have the form:

$$(Is - A)^{-1} = \sum_{j=0}^{n-1} s^j A_j N^{-1}(s),$$

$$N(s) = a_n s^n + \ldots + a_1 s + a_0,\ a_n = 1,$$

Assume also that the system is completely controllable and observable and for all eigenvalues of the matrix condition $\mathrm{Re}(\lambda_i + \lambda_j) \ne 0$ be true. Let us apply the identities (1) and (2) to computing the Gramian. Then we have(Yadykin (2010,2011)):

$$\mathbf{P} = \int_0^{\infty} \mathbf{e}^{\mathbf{A}\boldsymbol{\tau}} \mathbf{e}^{\mathbf{A}^{\mathrm{T}}\boldsymbol{\tau}} d\tau = \sum_{k=1}^{n} \sum_{j=0}^{n-1} \sum_{\eta=0}^{n_m-1} \frac{s_k^j (-s_k)^n}{N'(s_k) N(-s_k)} \mathbf{A}_{\mathbf{j}} \mathbf{A}_{\boldsymbol{\eta}}^{\mathbf{T}}, \tag{12}$$

Matrix $(\mathbf{I}s - \mathbf{A})$ determinant be represented as power system characteristic equation without considering functional characteristics, which could be computed by the local subsystems characteristic equations calculation. From theorem about determinant of the matrices sum arise, that determinant of matrix $(\mathbf{I}s - \mathbf{A}_1)$,being power system characteristic equation with considering functional characteristics, is a sum of matrix $(\mathbf{I}s - \mathbf{A})$ and sum of determinants, consisting of matrices $(\mathbf{I}s - \mathbf{A})$ and $\mathbf{M}^{-1}\mathbf{N}$ columns with admissible a cross between them.

The models of small signal stability calculation is fulfilled in state space form taking into account algebraic balance equations "on-line". The gramians with finite and infinite time are calculated by means of local subsystems models data usage (Yadykin (2010)). Frequency spectrums gramians allow to point out small signal stability indexes for local subsystems and central system in distributed computation scheme. The static stability indexes for local subsystems may be used in optimal adaptive coordination algorithms on the high level power control system.

Theorem 3 **(**Yadykin (2011)**).** Let a linear continuous stationary dynamic system be described by equations of the form (1) where the matrix A is real and Hurwitz, the roots s_k of the characteristic equation of the matrix A are plain and the conditions $s_k + s_\lambda \ne 0$, $\forall$k=1,2,..n, $\forall\lambda$=1,2,..n, are satisfied. Then the following limit relations are true:

i) If the root s_k of the characteristic equation is real and $\mathrm{Re}\, s_k = \alpha \to -0$, then

$$\lim_{Re\, s_k \to -0} P \sim \frac{1}{2\alpha} \frac{A_0 A_0^T}{N(-\alpha)(-s_2)\ldots(-s_n)} \tag{13}$$

ii) if the roots $s_k, s_{k+1} : s_k = j\omega_1 - \alpha, s_{k+1} = -j\omega_1 - \alpha,$ of the characteristic equation and $\mathrm{Re}\, s_k, s_{k+1} = -\alpha \to -0$, then