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Optimization analysis of convective–radiative longitudinal fins with temperature-dependent properties and different section shapes and materials

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ABSTRACT

The main aim of this study is to obtain an optimum design point for fin geometry, so that heat transfer rate reaches to a maximum value in a constant fin volume. Effect of fin thicknesses ratio (τ), convection coefficient power index (m), profile power parameter (n), base thickness (δ) and fin material are evaluated in the fin optimization point for heat transfer rate, effectiveness and efficiency. It's assumed that the thickness of longitudinal fins varies with length in a special profile, so four different shapes (rectangular, convex, triangular and concave) are considered. In present study, temperature-dependent heat generation, convection and radiation are considered and an analytical technique based on the least square method is proposed for the solution methodology. Results show that by increasing the fin thicknesses ratio, maximum heat transfer rate decreases and Copper among the other materials has the most heat transfer rate in a constant volume.

1. Introduction

Fins or extended surfaces have many applications in heat transfer such as air conditioning, refrigeration, automobile, chemical processing equipment and electrical chips for increasing the heat transfer rate. Recently, researchers and engineers in this field have been paying more attention to both energy conservation and equipment size reduction due to energy crisis in the world [1]. So, optimizing the heat transfer is highly desired for effective energy utilization. Because the weight and material cost of the extended surfaces are very important, fin dimensions should be optimized so that the least amount of fin material be used to dissipate a given amount of heat flow, or alternatively that the highest dissipation rate be obtained from a given volume of fin material. Also, using porous fins is another way for increasing the heat transfer in a constant volume of equipments which are introduced by Kiwan and Al-Nimr [2] and Kiwan [3] and many other researchers. Following some valuable researches in the fin optimizations are introduced.

Copiello and Fabbri [4] investigated the optimization of the heat transfer from wavy fins cooled by a laminar flow under conditions of forced convection. Authors in [4] considered the problem from a multi-objective point of view by finite element method (FEM) and genetic algorithm (GA) to find the geometries that maximize the heat transfer and minimize the hydraulic resistance at the same time. In another study, the optimum geometric dimensions of the parabolic profile circular fin with a constant volume, which yields the maximum heat dissipation, are determined by Xiang-xiang [5]. Also, Sharqawy and Zubair [6] found optimized dimensions for maximize heat transfer in a fully wet longitudinal fins considering heat and mass transfer.

In a different study, Kundu [7] considered a fin with a step reduction in cross section (SRC) and investigated the effect of various design and psychometric parameters on the fin performance of SRC fins and compared with the corresponding uniform cross section (UC) fin. Also, Kundu and Bhanja [8] designed a T-shape fin with maximum heat transfer by obtaining the lengths and thicknesses. Furthermore, Kundu et al. [9] obtained performance and optimum design of porous fin with various profiles operating in convection environment. In another study, Zhang and Chung [10] determined the optimal dimensions of a radiating convecting annular fin using an arbitrary profile and more specifically to present convenient design charts for the thermal designers. The heat transfer rate, effectiveness and efficiency for optimum fin design are obtained for this problem.

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Nomenclature

a	internal heat generation
c_i	coefficients of trial functions
$D()$	mathematical operator
h	convection heat transfer
h_b	convection heat transfer at base temperature
k	thermal conductivity
L	length of fin
m	parameter in temperature-dependent convection heat transfer
m_c	dimensionless parameter of convection heat transfer
m_r	dimensionless parameter of radiation heat transfer
n	parameter of fin profile
N	number of trial function
N_G	generation number
q	heat transfer
q_{actual}	actual heat transfer of fin
q_{ideal}	ideal heat transfer
q_p	heat dissipation without fin
q_G	internal heat generation
Q	dimensionless heat transfer
$R(x)$	residual function
S	summation of squared residual
T	temperature
T_a	temperature of surrounding fluid
T_b	base temperature of fin
T_s	sink temperature for radiation heat transfer
$t(x)$	fin profile
$\tilde{u}$	trial function
V	volume of fin
W_i	weight function
X	horizontal direction

Greek symbols

ε	surface emissivity
ε_s	surface emissivity at the temperature of radiation sink
η	fin efficiency
θ	dimensionless temperature
σ	Stefan–Boltzmann constant
τ	dimensionless parameter of thickness
ω	dimensionless parameter in heat generation
ζ	fin effectiveness

Turkyilmazoglu [11] found a good solution for thermal diffusion in a straight fin with varying exponential shape when the thermal conductivity and heat transfer coefficients are temperature dependent by power laws. An important result of Turkyilmazoglu [11] is that the efficiency and base heat transfer rate of the exponential profiles are higher than those of the rectangular fin. Aziz and Beers-Green [12] investigated the performance and optimum design of a longitudinal rectangular fin attached to a convectively heated wall of finite thickness by numerical method obtained by Maple package. The results in [12] are compared their results by those obtained by Adomian's decomposition and the differential quadrature method. Khani and Aziz [13] applied Homotopy Analysis Method (HAM) for predicting the thermal performance of a straight fin of trapezoidal profile when both the thermal conductivity and the heat transfer coefficient are temperature dependent.

Thermal behavior of the fins with the power-law temperature-dependent thermal conductivity and heat transfer coefficient are considered by Mosayebidorcheh et al. [14] through Differential Transformation Method (DTM) considering all nonlinearities. Also Joneidi et al. [15] and Mosayebidorcheh and Mosayebidorcheh [16] used the same method to determine the fin efficiency of convective straight fin and Ganji et al. [17] used HPM analytical method for thermal analysis of annual fins. Another high accuracy analytical method which is used for solving the problems, is Least Square Method (LSM) which is widely used by authors [18–24] in solving different engineering problems such as fins thermal analysis. Hatami et al. [25] presented a review of the technologies that increase the heat transfer rate in heat exchangers and proposed some points to raise heat recovery from the exhaust of diesel engines.

Razelos and Imre [26] investigated the effects of curvature and thermal properties of the convective circular fin. They considered three profiles: rectangular, triangular, and trapezoidal. Ullmann and Kalman [27] studied the convective annular fin of triangular, hyperbolic, rectangular and parabolic shapes. They obtained the fin efficiencies and the optimum dimensions for these four different radial fin profiles. Zubair et al. [28] found the optimum circular fin dimensions with variable profile considering temperature-dependent thermal conductivity. Chung and Ma [29] considered the wall thermal resistance effect on the convective fins.

Motivated by above mentioned works, in present study authors aim to use LSM for obtaining the temperature distribution in longitudinal fins with different section shapes and heat generation. The approximate solution of problem is compared to the numerical solution of governing equations which Also, a complete optimization study for fin geometry and material is performed to reach an optimum point for the maximum heat transfer rate, effectiveness and fin efficiency. The novelties of this work can be defined as follows:

- Introducing an optimum design for a fin with variable sections based on effectiveness and efficiency.
- Considering different material in optimum analysis.
- Using a powerful semi-analytical method for solving the problem directly.

2. Mathematical analysis

2.1. Governing equations

Consider a one-dimension longitudinal fins of rectangular, convex parabolic, triangular and concave parabolic profiles with the length L as shown in Fig. 1. The heat transfer from the fin surface is both convection and radiation. Suppose the effective sink temperature for radiative heat transfer is T_s and the temperature of surrounding fluid is T_a. The heat transfer of the fin tip is assumed negligible. The convective heat transfer h and surface emissivity ε are considered to be functions of temperature of forms [14]

$$h = h_b\left(\frac{T - T_a}{T_b - T_a}\right)^m \quad (1)$$

$$\varepsilon = \varepsilon_s[1 + \beta(T - T_s)] \quad (2)$$

where T_b is the base temperature of fin, h_b is the convection heat transfer at the temperature T_b, ε_s is the surface emissivity at the temperature of radiation sink T_s and m and β are constants. The constants m and β are measures of variation of convection heat transfer and surface emissivity with temperature, respectively. The power m in Eq. (1) depends on the heat transfer mode. Typical values of m are $-1/4$ for condensation or laminar film boiling, $1/4$ for laminar nat-