Energy Conversion and Management 106 (2015) 300–307

Contents lists available at ScienceDirect

Energy Conversion and Management

journal homepage: www.elsevier.com/locate/enconman

A synthetic layout optimization of discrete heat sources flush mounted on a laminar flow cooled flat plate based on the constructal law

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ARTICLE INFO

Article history:
Received 29 June 2015
Accepted 21 September 2015

Keywords:
Discrete heat sources
Constructal law
Laminar flow
Flat plate
Pareto optimization
Surface temperature control
Flow drag

ABSTRACT

A synthetic optimization is presented for the Pareto layouts of discrete heat sources (with uniform heat flux) flush mounted on a flat plate over which laminar flow serves for cooling purpose. The peak temperatures and the flow drag loss are minimizing simultaneously provided that the total heat dissipation rate and the plate length are held constant. The impact of the manufacturing limit, i.e. the minimum length of the heated or the adiabatic patch, on the optimum layout is discussed. The results in general comply with analytical deduction based on the constructal theory. However in a finite length scenario, geometric constraints on the adiabatic spacing differ from that fits the situation in which maximum heat transfer performance alone is to be achieved.

1. Introduction

Design and optimization of applications in which convective heat transfer dominates has been an enduring field of study, particularly for those with demandingly high rate of heat transfer in a limited space to ensure a stable system performance, e.g. hydrogen supplying for fuel cell [1], gas turbine cooling [2], high-power LED cooling [3,4]. For flow boiling, the heat transfer is mechanistically intertwined with interfacial effect [5], flow pattern [6] and so on and is therefore more vulnerable to drawbacks such as bubble clogging and local superheating [7]. Whereas convective heat transfer, with either active [8,9] or passive [10] enhancement, is gaining more attention with a catching-up heat flux.

The investigation concerning the heat transfer and flow resistance characteristics of discrete heat sources flush-mounted on a flat plate, which benefits from the non-intrusive design with less pressure head loss in contrast to the fin structure, has been an ongoing topic in the design of high power density stacks with respect to thermal management for several decades. Incropera et al. [11] conducted experiments for flush-mounted discrete heat sources on a wall of a horizontal channel. Convective heat transfer was investigated and compared between the single-heat-source case and the case with 12 heat sources in a 3-by-4 rectangular array (4 rows in the flow direction), using water and FC-77 as the working fluids. The Reynolds number was within a range of 1000–14,000. The convection coefficient of single-heat-source case resembled that of the first-row, following which a reduction with successive descending rates – 25%, 10% and 5% were reported when streamwise comparing two consecutive rows. An under-prediction in heat transfer performance was also mentioned from the comparison with the two-dimensional, steady laminar flow with constant fluid properties, whereas a good agreement for the turbulent flow excluded the possible effect from the boundary layer, and the substrate conduction as well. Tso et al. [12] reported the experimental results for four flush mounted heat sources aligned in the flow direction of cooling water, which is pumped vertically upward. Thermally, it was illustrated that the influences from Reynolds number and location of heat sources are more prominent than that from channel height, which is negligibly weak resembling the external flow case. Whereas the hydraulic diameter of the channel turned out more suitable for characterizing the flow regime, which is in nature more of internal flow.

The performance optimization regarding the geometric layout started receiving vast attention in electronics cooling [13,14]. Morega and Bejan [15] investigated the thermohydraulic performance

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http://dx.doi.org/10.1016/j.enconman.2015.09.057
0196-8904/

Nomenclature

$_2F_1$	Gauss hypergeometric function of order 1, 2
a	constant for the Gauss hypergeometric function, see Eqs. (9) and (10)
b	constant for the Gauss hypergeometric function, see Eqs. (9) and (10)
c	constant for the Gauss hypergeometric function, see Eqs. (9) and (10)
i	variable for the Gauss hypergeometric function, see Eq. (10)
k	fluid thermal conductivity of the coolant fluid, W/(m K)
l	length, for either the heated or the adiabatic segment, m
L	total length of the plate, m
n	total number of heat sources
N	variable for the Gauss hypergeometric function, see Eqs. (9) and (10)
Pr	Prandtl number of the coolant fluid
q''	local heat flux along the plate, W/m^2
$\dot{q}$	magnitude of the uniform heat flux, W/m^2
Re	Reynolds number of the coolant fluid
T	temperature, K
x	abscissa, indicating the location on the plate, m
y	variable for the Gauss hypergeometric function, see Eq. (10)
z	variable for the Gauss hypergeometric function, see Eq. (9)

Greek symbols

ξ	dummy variable for integration, see Eqs. (4), (7) and (8), m
σ	distribution function of the local heat flux

Superscripts and subscripts

$\sim$	indicating the variable being nondimensionalized, see Eq. (5)
i	index for the heated or the adiabatic segments, see Fig. 1
∞	free stream
max	maximum
min	minimum
q	heat flux
s	surface
x	local value at abscissa x

of the discrete heat sources flush mounted at the bottom wall of the channel formed between two parallel plates. The top wall and the remaining part of the bottom wall are adiabatic. All the five heat sources are under an on–off control individually, with a uniform heat flux in common and a total heat generation rate held constant for all cases in comparison. The location of a single heat source in charge was found exerting little impact on the maximum surface temperature. A characteristic length, summing up the total length of all heat sources, including the adiabatic patches in between, was proposed. The maximum thermal conductance (dimensionless) the was plotted against the parameter consisting of the channel aspect ratio and the non-dimensional pressure drop, for the comparison between cases with 1, 3, 5 heat sources located at the leading edge. Da Silva et al. [16] reported their investigation on maximizing the thermal conductance between the coolant fluid and the discretely heated wall, which is equivalent to minimizing the "hot spot" temperature. The optimal placement of the discrete heat sources with finite size are expected to be non-uniform, with the heat source of largest size located at the leading edge. An increasing distance was identified between each pair of the neighboring heat sources downstream. Cases with up to three heat sources were compared, optimal spacings were found highly sensitive to the variation of the Reynolds number and the heat source length. Cases of more heat sources with optimal spacings yielded higher thermal conductance. Breaking through the theoretical constraints [16] on both sizes and spacings of a finite number of heat sources, Hajmohammadi et al. [17] compared the results from the minimization of "hot spots" temperature for cases with and without limits on the size and cooling load of the entire plate. Degrees of morphing freedom again was found having a significant influence on the minimized "hot spot" temperature.

However, a synthetic optimization for parameters regarding temperature peak suppression, flow friction, lower size limit in manufacturing practice and so forth is in lack of consideration after reviewing the extant literature of relevance. In the present study, the constructal theory was applied to achieve the optimal geometric configuration of a staggered set of heat sources and adiabatic segments. The thermal resistance and the fluid flow resistance were minimized simultaneously, the overall heat transfer rate and the total plate length held invariant. The impacts from different dimensions of the morphing freedom are discussed concerning the minimum manufacture limit, the total number of heat sources and the constraints on the heat source length and the in-between spacing, etc. The conclusion could potentially serve as a guideline for the design that represents superior thermohydraulic performance.

2. Model description

2.1. Analytical model

As depicted in Fig. 1, the free stream with the temperature, denoted as T_∞, and the uniform flow velocity, denoted as U_∞, comes in contact with the flat plat surface, on which the flush amounted heat source (with uniform heat flux) corresponds to a continuous surface temperature rise, streamwise with a decreasing slope. The adjacent patch with zero heat flux, on the contrary, leads to a decline in surface temperature. A cliff-style drop, from where the adiabatic segment starts, is followed by a gradually flattening curvature after which a second turn cycle begins forming this Sydney-Opera-House-shaped tandem. The following procedure illustrates how the temperature variation is analytically obtained.

All the discrete heat sources are of uniform heat flux. Thus the local heat flux along the abscissa can be expressed as

$$q''(x) = \dot{q} \cdot \sigma_q(x) \tag{1}$$

where

$$\sigma_q(x) = \begin{cases} 0, & x_i - l_{a,i} < x < x_i \\ 1, & x_i - (l_{h,i} + l_{a,i}) < x < x_i - l_{a,i} \end{cases}, \quad i = 1, 2, \ldots, n \tag{2}$$

in which

$$x_i = \sum_{k=1}^{i} l_{a,i} + \sum_{k=1}^{i} l_{h,i}, \quad i = 1, 2, \ldots, n \tag{3}$$

The surface temperature variation then follows the similarity solution for the cases of non-uniform heat flux in general [14]

$$T_s(x) - T_\infty = \frac{0.623}{k} Pr^{-1/3} Re_x^{-1/2} \int_{\xi=0}^{x} \left[1 - \left(\frac{\xi}{x}\right)^{3/4}\right]^{-2/3} q''(\xi) d\xi \tag{4}$$