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Heat transfer and thermodynamic performance of convective–radiative cooling double layer walls with temperature-dependent thermal conductivity and internal heat generation

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ABSTRACT

Composite geometries have numerous applications in industry and scientific researches. This work investigates the temperature distribution, and local and total entropy generation rates within two-layer composite walls using conjugate convection and radiation boundary conditions. Thermal conductivities of the materials of walls are assumed temperature-dependent. Temperature-dependent internal heat generations are also incorporated into the modeling. The differential transformation method (DTM) is used as an analytical technique to tackle the highly nonlinear system of ordinary differential equations. Thereafter, the local and total entropy generation rates are calculated using the DTM formulated temperature distribution. An exact analytical solution, for the temperature-independent model without radiation effect, is also derived. The correctness and accuracy of the DTM solution are checked against the exact solution. After verification, effects of thermophysical parameters such as location of the interface, convection–conduction parameters, radiation–conduction parameters, and internal heat generations, on the temperature distribution, and both local and total entropy generation rates are examined. To deliver the minimum total entropy generation rate, optimum values for some parameters are also found. Since composite walls are widely used in many fields, the abovementioned investigation is a beneficial tool for many engineering industries and scientific fields to minimize the entropy generation, which is the exergy destruction, of the system.

1. Introduction

Composite walls have attracted considerable attentions due to their enhanced thermophysical properties. For example, a composite wall can be used in building industries as a good insulator [1–4]. Therefore, they have been intensively analyzed from the heat transfer point-of-view. However, the heat transfer perspective, i.e., the first law of thermodynamics, is a tool to quantitatively investigate the thermal performance of these structures. To qualitatively look through the thermal performance of these composite structures, one needs another tool, which is the second law of thermodynamics. The second law of thermodynamics gives researchers an invaluable tool to examine the performance of any thermal system regarding the entropy generation, which is a measure of the available exergy destruction and consequently the irreversibility of the system [5–8].

Conduction [9,10], convection [11–15] and radiation [16] are three models of heat transfer, which produce entropy. Apart from them, in many thermal systems, viscous effects [13,14] and magnetic fields [17,18] are two other important factors in the entropy generation. As discussed by Bejan [19,20], the entropy generation within a steady state system is always nonnegative and constant. In Bejan's treatise [20], it was systematically discussed that the entropy of a steady state system can be optimized using the entropy generation minimization (EGM) approach. Therefore, using the EGM concept, many researchers began to reexamine thermal systems from the second law point-of-view. Natural and forced convections in different geometries are two main subjects that have been evaluated from the second law perspective [21,22]. However, there is a dearth of literature regarding the second law of thermodynamics in pure conductive media with convective, radiative or convective–radiative boundary conditions.

Entropy generation within conductive media depends on the temperature distribution through the system [9]. The temperature distribution within a solid medium is mainly governed by thermophysical properties of the medium such as thermal conductivity,

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Nomenclature

d_1	slope of the thermal conductivity-temperature curve for inner material, K^{-1}
d_2	slope of the thermal conductivity-temperature curve for outer material, K^{-1}
h_1	convection heat transfer coefficient at the inner surface, $W\,m^{-2}\,K^{-1}$
h_2	convection heat transfer coefficient at the outer surface, $W\,m^{-2}\,K^{-1}$
k_1	reference thermal conductivity for inner material, $W\,m^{-1}\,K^{-1}$
k_2	reference thermal conductivity for outer material, $W\,m^{-1}\,K^{-1}$
k_r	thermal conductivities ratio
L	total thickness of the wall, *m*
N_{c1}	convection–conduction parameter at the inner surface
N_{c2}	convection–conduction parameter at the outer surface
Nr_1	radiation–conduction parameter at the inner surface
Nr_2	radiation–conduction parameter at the outer surface
Q_1	dimensionless volumetric internal heat generation rate for the inner material
Q_2	dimensionless volumetric internal heat generation rate for the outer material
$\dot{q}_1$	volumetric internal heat generation rate for the inner material, $W\,m^{-3}$
$\dot{q}_2$	volumetric internal heat generation rate for the outer material, $W\,m^{-3}$
T	temperature, K
T_1	temperature of inner material, K
T_2	temperature of outer material, K
T_L	sink temperature at inner side, K
T_R	sink temperature at outer side, K
X	dimensionless axial distance
ζ	dimensionless interface distance
x	axial distance, m
x_i	interface distance, m

Greek symbols

λ_1	dimensionless slope of the thermal conductivity-temperature curve for inner material
λ_2	dimensionless slope of the thermal conductivity-temperature curve for outer material
ε_1	emissivity of the inner surface
ε_2	emissivity of the outer surface
θ	dimensionless temperature
θ_1	dimensionless temperature of inner material
θ_2	dimensionless temperature of outer material
θ_R	ratio of the inner sink to outer sink temperatures
σ	Stefan–Boltzmann constant, $W\,m^{-2}\,K^{-4}$

aspect ratios within a system, internal heat generation and imposed boundary conditions. When dealing with heat transfer in conductive media, an important simplification is that the temperature variation is unidirectional. Considering this assumption, the partial differential energy equation of the solid is changed into an ordinary differential equation (ODE). The obtained ODE can be easily solved numerically or, in some specific cases, analytically. Bisio [23,24] is one of the pioneers to analyze the entropy generation in pure conductive media. Following Bisio's work [23,24], Ibáñez et al. [25] analyzed the entropy generation rate in slabs with constant internal heat generation and asymmetric convective heat transfer on boundary surfaces. They pointed out that choosing proper convective properties, i.e., optimum convection heat transfer rates, the entropy generation within the solid medium can be minimized. Kolenda et al. [26] extended Ibáñez et al.'s study [25] to a multidimensional analysis. Bautista et al. [27] extended Ibáñez et al.'s study [25] to a transient entropy generation in one-dimensional slabs having internal heat generation. They however considered symmetric convective cooling on exposed surfaces.

Although constant thermophysical properties are often assumed in modellings, most of properties and natural phenomena are inherently variable, i.e., non-constant, and therefore nonlinear. Sahin [28] calculated the entropy generation in steady state heat conduction through a slab with or without internal heat generation. Temperature-dependent thermal properties were also considered in this study. Assad [29] opted in favor of entropy generation within slabs with location-dependent internal heat generation. Due to laser irradiation, it was assumed that the internal heat generation varies exponentially with space. Temperature distribution in slabs with temperature-dependent internal heat generation was analytically studied by Aziz and Khan [30]. They assumed that the slab was asymmetrically cooled with convection. After analytical solution for the temperature distribution, the local and total entropy generation rates for number of cases were also plotted. They repeated these calculations within hollow spheres with temperature-dependent internal heat generation and constant thermal conductivity [31]. In an interesting work, Aziz and Khan [32] analytically investigated the classical and minimum entropy generation within three geometries. They modeled the temperature and entropy generation within slabs, and hollow cylinders and spheres, using homogenous or functionally graded materials. Torabi and Aziz [9] extended the previous work of Aziz and Khan [32] to hollow cylinders with convective–radiative cooling condition on the outer surface. They also incorporated temperature-dependent internal heat generation within the energy equation. They used the differential transformation method (DTM) to analytically address the nonlinear energy equation of the geometry. Later, Torabi and Zhang [10] examined homogenous and functionally graded convective–radiative slabs with temperature-dependent internal heat generation from the second law perspective.

As mentioned earlier, composite and multilayer structures have various applications in energy systems. Their superb advantages, such as thermal insulation [1,3,33,34], have led research communities to drastically use them in various fields. Applications of composite materials can be found in energy systems, bioengineering, packaging, civil and nuclear engineering, etc. [35–39]. With a view to the practical applications of these structures, accurate information regarding the temperature distribution through these structures is essential. Moreover, using the obtained temperature distribution, the local and total entropy generation rates can be achieved with the aid of available analytical or numerical techniques. Few of entropy generation studies in pure conductive media deal with multilayer geometries. Recently, Torabi and Zhang [40] have solved the problem of entropy generation within double layer hollow cylinders. They have used a combined analytical–numerical method to tackle the system of ODEs. Two types of boundary conditions were selected. The first case dealt with constant temperature boundary condition and the second one dealt with convective boundary conditions on both inside and outside surfaces. However, entropy generation within double layer walls with convective–radiative boundary conditions has not been investigated yet. A deep insight into the important factors and at