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Effect of heat source on transient energy growth analysis of a thermoacoustic system

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ABSTRACT

Transient growth of disturbances in a non-normal combustion system could trigger thermoacoustic instability. In this work, the effect of heat source on triggering thermoacoustic instability in a system with Dirichlet or Neumann boundary conditions is investigated. For this a thermoacoustic model of a premixed laminar flame is developed. It is formulated in state-space by expanding flow disturbances via Galerkin series and coupling with a flame model, thus providing a platform on which to gain insight on the system stability behaviors. Transient energy growth analysis is then performed by discretizing the flame model into distributed monopole-like sound sources. It is shown that the thermoacoustic system is non-normal and associated with transient energy growth. Parametric studies are then conducted to study the effects of (1) the flame-acoustics interaction index $\mathscr{I}$, (2) the eigenmode number N, (3) the mean temperature ratio $\mathscr{T}$ between pre- and after-combustion regions on the system stability behavior and non-normality. It is found that the maximum transient energy growth G_{max} is increased with increased $\mathscr{I}$. Furthermore, the thermoacoustic system with more eigenmodes is found to be associated with a larger G_{max}.

1. Introduction

To achieve low NO_x emission and increased combustion efficiency, lean premixed pre-vaporized combustion technology is widely applied in aero-engine and gas turbines. However, they are more susceptible to thermoacoustic instability [1–4]. It occurs frequently in other combustion systems such as rocket motors, ramjets, boilers or furnaces. Such instability is generated due to the coupling between unsteady combustion process and acoustic disturbances present [5]. As an efficient monopole-like sound source, unsteady combustion produces acoustic waves [6]. And these pressure waves [4] propagate within the combustor and partially reflect from boundaries to arrive back at the combustion zone. When unsteady heat is added in phase with the pressure oscillations [7], acoustical energy increases until limit cycle oscillations [8] occur. Such oscillations are wanted in thermoacoustic engine systems [9–19]. However, they are undesirable in aero-engines and gas turbines, since the oscillations may be so intensive that they can cause structural damage, enhanced heat transfer and costly mission failure to the engine systems [5,20].

Thermoacoustic systems have been shown to be nonlinear [21,22] and non-normal [23–29]. The non-normality [26] is characterized by non-orthogonal eigenmodes. It arises from unsteady heat release and/or the complex impedance boundary conditions. It has also been shown that in a linearly stable but non-normal thermoacoustic system, there can be significant transient energy growth of small perturbations before their eventual decay. If the disturbance transient growth is large enough, thermoacoustic instability might be triggered by causing such disturbances to grow to amplitudes high enough to make nonlinear effects significant. However, different energy measures are used in the literature to characterize transient growth [24,30–33].

Chu [31] argued that the disturbance energy in a viscous compressible flow should include the entropy fluctuations in addition to conventional acoustical energy, which consists of both pressure and velocity fluctuations. Morfey [32] suggested that the total disturbance energy should include the energy associated with fluctuations in vorticity and entropy, beside the classical acoustical energy. Nicoud and Poinsot [34] argued that the classical Rayleigh criterion is incomplete in describing the significant sources of fluctuating energy in a combustion system with a flow. Giauque et al. [35] relaxed the definition of the disturbance energy from Myers by considering chemical species and heat release terms. Initially, the energy measure used to study the transient growth in thermoacoustic system included the acoustical energy only [24,25,30]. However, there is no consensus on the definition of the disturbance energy. In addition to acoustical energy, entropy

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fluctuations were introduced by Mariappan and Sujith [23] in the energy measure. Subramanian and Sujith [33] introduced the unsteady heat release fluctuations in the energy measure. However, the previous transient growth analysis was conducted by assuming there is no temperature difference between the pre- and after-combustion regions. Thus there is a need to investigate the mean temperature effect on quantifying the disturbance transient growth. Lack of such investigations partially motivated the present work.

Experimental investigations were performed to measure the transient growth rate of thermoacoustic systems. Mariappan et al conducted such transient growth measurements on a Rijke tube [28], which is a simple but widely used platform [36–39] to study heat-to-sound conversion. Kim and Hochgreb [41] performed similar measurements on a lean-premixed gas turbine combustor in Cambridge. The transient energy growth of the disturbances due to the non-normality cannot be predicted by the classical linear stability theory [21], since it provides information only about the long-term evolution of the eigenmodes [1,27,29]. To eliminate the disturbance transient growth, classical linear controllers are not applicable [27,29,40].

In this work, a simplified thermoacoustic system with Dirichlet or Neumann boundary conditions [42] is considered. A premixed laminar flame is confined and leads to a mean temperature jump from pre- to after-combustion region. In Section 2, the system equations are developed in state-space. The flow perturbations are expanded by using Galerkin series and the flame front is tracked by using the classical *G*-equation [43,44]. And unsteady heat release from the flame is assumed to be caused by its surface variation, which results from the fluctuations of the oncoming flow velocity. Both nonlinear and linearized flame models are discussed. In Section 3, transient energy growth analysis of the thermoacoustic system is performed. The energy measure consists of both acoustical energy and the unsteady heat release fluctuations by discretizing the flame front into distributed monopole-like sound sources. In Section 4, the results are presented and the key findings are discussed.

2. Description of the thermoacoustic model

In the present work, we consider a simplified combustion system with a monopole-like flame and Dirichlet boundary conditions implemented as shown in Fig. 1.

A premixed laminar conical-shaped flame acts as an acoustically compact heat source. It is modeled as a thin sheet and described by using Dirac delta function in mathematics. The flow variables with a tilde are the instantaneous quantities, which consist of a mean part denoted with an overbar and a fluctuating part. The dimensional governing equations of the thermoacoustic systems comprise:

$$\bar{\rho}\frac{\partial u}{\partial t}+\frac{\partial p}{\partial x}=0 \tag{1}$$

$$\frac{1}{\bar{c}^2}\frac{\partial^2 p}{\partial t^2}+\frac{\zeta}{\bar{c}^2}\frac{\partial p}{\partial t}-\frac{\partial^2 p}{\partial x^2}=\frac{(\gamma-1)}{\bar{c}^2}\frac{\partial}{\partial t}\left(Q_f(t)\delta(x-x_f)\right) \tag{2}$$

where $\bar{c}$ is the sound speed, L_0 is the length of the duct, γ is the ratio of specific heats, $\delta(x)$ is dirac delta function describing the localized flame or the actuator, Q_f denotes the unsteady heat release from the flame.

Expanding the pressure perturbation as Galerkin series [25,27,30] gives,

$$p(x,t)=\sum_{n=1}^{N}\frac{-1}{\kappa_n\omega_n}\dot{\eta}_n(t)\psi_n(x) \tag{3}$$

where

$$\kappa_n=\left[\int_0^{x_f}\psi_n^2(x)\mathrm{d}x+\int_{x_f}^{L_0}\psi_n^2(x)\mathrm{d}x\right]^{1/2} \tag{4}$$

N denotes the eigenmode number. Overdot denotes the time derivative. The function $\psi_n(x)$ are the eigen-solutions of the homogeneous wave equation,

$$\frac{1}{\bar{c}^2}\frac{\partial^2 p}{\partial t^2}-\frac{\partial^2 p}{\partial x^2}=0 \tag{5}$$

Since the presence of the flame results in the mean temperature undergoing a jump from the region $0\leqslant x<x_f$ to $x_f<x\leqslant L_0$, substituting Eq. (3) into Eq. (5) gives

$$\frac{\omega_n^2}{\bar{c}_1^2}\psi_n+\frac{\mathrm{d}^2\psi_n}{\mathrm{d}x^2}=0,\ \ 0\leqslant x<x_f\ \ \text{and}\ \ \frac{\omega_n^2}{\bar{c}_2^2}\psi_n+\frac{\mathrm{d}^2\psi_n}{\mathrm{d}x^2}=0,\ \ x_f<x\leqslant L_0 \tag{6}$$

Applying the pressure continuity and velocity jump across the flame and the Dirichlet boundary conditions at $x=0$ and $x=L_0$ [25,27,30], i.e. $p(x,t)|_{x=0}=0$ and $p(x,t)|_{x=L_0}=0$ leads to

$$\psi_n(x)=\begin{cases}\sin\left(\frac{\omega_n x}{\bar{c}_1}\right) & 0\leqslant x<x_f\\ -\sin\left(\frac{\omega_n(x-L_0)}{\bar{c}_2}\right)\frac{\sin\left(\omega_{n}x_f/\bar{c}_1\right)}{\sin\left(\omega_n(L_0-x_f)/\bar{c}_2\right)} & x_f<x\leqslant L_0\end{cases} \tag{7}$$

that satisfies the condition of orthogonality, i.e. $\int_0^{L_0}\psi_m(x)\psi_n(x)\mathrm{d}x=0$ for $m\neq n$. Further analysis illustrates that ω_n is related to x_f in the form of

$$\bar{c}_2\sin\left(\frac{\omega_n x_f}{\bar{c}_1}\right)\cos\left(\frac{\omega_n(L_0-x_f)}{\bar{c}_2}\right)=-\bar{c}_1\sin\left(\frac{\omega_n(L_0-x_f)}{\bar{c}_2}\right)\cos\left(\frac{\omega_n x_f}{\bar{c}_1}\right) \tag{8}$$

Eq. (8) can be used to characterize the frequency ω_n of the eigenmode (see Fig. 2).

Substituting Eq. (7) into Eq. (3) yields

$$p(x,t)=\begin{cases}-\sum_{n=1}^{N}\frac{\dot{\eta}_n(t)}{\kappa_n\omega_n}\sin\left(\frac{\omega_n x}{\bar{c}_1}\right) & 0\leqslant x<x_f\\ \sum_{n=1}^{N}\frac{\dot{\eta}_n(t)}{\kappa_n\omega_n}\sin\left(\frac{\omega_n(x-L_0)}{\bar{c}_2}\right)\frac{\sin\left(\omega_n x_f/\bar{c}_1\right)}{\sin\left(\omega_n(L_0-x_f)/\bar{c}_2\right)} & x_f<x\leqslant L_0\end{cases} \tag{9}$$

Replacing $p(x,t)$ in the momentum Eq. (1) with Eq. (9) gives

$$u(x,t)=\begin{cases}\sum_{n=1}^{N}\frac{\eta_n(t)}{\bar{\rho}_1\bar{c}_1\kappa_n}\cos\left(\frac{\omega_n x}{\bar{c}_1}\right) & 0\leqslant x<x_f\\ -\sum_{n=1}^{N}\frac{\eta_n(t)}{\bar{\rho}_2\bar{c}_2\kappa_n}\cos\left(\frac{\omega_n(L_0-x)}{\bar{c}_2}\right)\frac{\sin\left(\omega_n x_f/\bar{c}_1\right)}{\sin\left(\omega_n(L_0-x_f)/\bar{c}_2\right)} & x_f<x\leqslant L_0\end{cases} \tag{10}$$

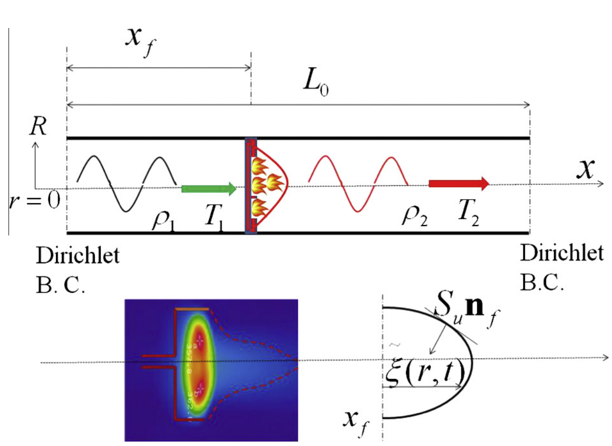

Fig. 1. Schematic of the combustion system with an acoustically compact premixed flame confined and Dirichlet boundary conditions applied on both inlet and outlet.