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Transient thermal analysis of longitudinal fins with internal heat generation considering temperature-dependent properties and different fin profiles

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ABSTRACT

The present paper aims to study the transient thermal analysis of longitudinal fins with variable cross section considering internal heat generation. The profile shapes of the fins are considered rectangular, convex, triangular and concave. It is assumed that both thermal conductivity and internal heat generation are as linear functions of temperature. The power-law temperature-dependent model is used to simulate different types of heat transfer such as laminar film boiling, natural convection, nucleate boiling and radiation. The governing equation is derived as a nonlinear partial differential equation (PDE) that is solved using a hybrid approximate technique based on the differential transform method (DTM) and finite difference method (FDM). The results are presented to study the effects of some physical parameters such as fin profile shape, thermal conductivity, convection heat transfer coefficient and internal heat generation.

1. Introduction

Heat transfer has an important role in various engineering problems. The extended surfaces or fins have a significant role in the enhancement of heat transfer from the surfaces. Fins have some applications in small systems such as electronic components and transistors; and large systems such as industrial heat exchangers. A general review on the extended surfaces is reported by Kraus et al. [1]. Heat transfer analysis of the fins has been conducted by numerous scientists and engineers. A significant assumption made in the analysis of fins is based on the simplification that variation of the temperature in the lateral direction is negligible. It means that the temperature at any cross section of the fin is constant. This assumption simplifies the governing equation of the fins since it not only converts the mathematical formulation for steady state from partial differential equation (PDE) to ordinary differential equation (ODE), so it can be obtained an analytical solution of the problem for a number of cases.

Most of the scientific and engineering phenomena such as heat transfer problems are naturally nonlinear. Usually some assumptions are used to transform the nonlinear problems to linear cases. For example, the thermo-physical properties are considered as constant parameters to obtain the temperature distribution of the extended surfaces. If a great temperature difference occurs in the fin, the thermo-physical parameters are not constant [2]. So in general form, the thermo-physical parameters such as thermal conductivity and convection heat transfer coefficient are functions of temperature.

Recently, some analytical and approximate methods have been used by researchers to solve the nonlinear heat transfer equation of the fins. Also, the thermal analysis of the radial and straight fins Considering temperature-dependent thermal conductivity is investigated by the variational iteration method (VIM) [3,4]. Domairry and Fazeli [5] studied the nonlinear differential equation of straight fins by homotopy analysis method (HAM) to obtain the temperature distribution within the fins. The optimum design of an annular fin with temperature-dependent properties is analyzed by Arslanturk [6]. In another work, Kulkarni and Joglekar [7] utilized a numerical method based on residue minimization to solve the nonlinear differential equation of straight convective fins having temperature-dependent thermal conductivity. In [8], differential transform method (DTM) is used to evaluate the analytical solution of the nonlinear fin problem with temperature-dependent thermal conductivity. The thermal conductivity of the fins is usually approximated by a linear temperature-dependent relation [9–10].

Generally, the convection coefficient of the heat transfer may vary along the fins. This coefficient may be a function of local

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temperature difference between the surrounding fluid and fin surface [11] or may depend on spatial coordinate only along a fin [1]. In general form, the convection coefficient h is a nonlinear function of the local temperature difference ΔT. Many convective heat transfer processes such as radiation, boiling and natural convection, happening on the surface of the fins can be adequately exhibited by a power-law type of dependence $h \propto (\Delta T)^m$. Aziz and Torabi [12] considered a general form of the fin problem. They studied the convective–radiative straight fins having temperature-dependent thermal conductivity, heat transfer coefficient and surface emissivity. They also investigated the thermal performance and efficiency of T-shaped fins considering variation of properties for both stem and flange parts with temperature [13]. The thermal performance of the longitudinal fins of rectangular, trapezoidal and concave parabolic profiles is investigated [11]. In this work, thermal conductivity, convection coefficient and surface emissivity are considered as a function of temperature. Aziz et al. [14] studied the convective–radiative radial fins for homogeneous and functionally graded materials (FGM). The boundary conditions of their analysis are considered the convective base heating and convective–radiative tip cooling. Hatami and Ganji [15] have presented the thermal analysis of circular convective–radiative porous fins with different section shapes and materials. Also, heat transfer study through porous fins (Si_3N_4 and Al) considering temperature-dependent heat generation is investigated by Hatami et al. [16]. They have used analytical methods, DTM, collocation method (CM) and least square method (LSM) to predict the temperature distribution in the porous fins.

Although designs of systems employing extended surfaces are based on steady-state analyses, which are adequate for most applications, there are situations in which knowledge of the transient response is necessary. This is true for fins used in high-speed aircraft, intermittently operating heat exchangers, electronic components, automatic control equipment, and solar energy systems [1]. Recently, transient response of the longitudinal rectangular fins was investigated using the symmetry analysis [17,18]. Authors in [18] considered the step change and heat flow conditions at the fin base and the effects of the thermo-geometric fin parameters and temperature-dependent properties were discussed.

In this paper, thermal analysis of the longitudinal fins with variable cross-sectional area and temperature-dependent thermal conductivity, heat transfer coefficient and internal heat generation is investigated. The cross section profiles of the fins are considered to be rectangular, convex, triangular and concave. Thermal conductivity and internal heat generation are regarded as linear functions of temperature, while convection coefficient is considered as a power-law function of temperature.

In this work, DTM is used to solve the nonlinear PDE of the described problem. DTM was first presented by Zhou [19] to solve the initial value problems (IVPs) in the analysis of electrical circuit. This method has been applied to some computational and engineering problems such as boundary value problems (BVPs) [20] advective–dispersive transport problem [21], nonlinear heat transfer of fins [22,23], Strum-Liouville equation [24] and vibration problem [25]. Here, a hybrid numerical algorithm which combines DTM and finite difference method (FDM) is utilized to study the present problem. The DTM and FDM are applied on the time and space domains of the problem, respectively.

2. Differential transform method

The differential transform is defined as follows

$$X(k) = \frac{H^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right] \tag{1}$$

where $x(t)$ is an analytical function in the time domain, H is the time interval and $X(k)$ is the transformed function. The inverse transformation is as follows

$$x(t) = \sum_{k=0}^{\infty} X(k)\left(\frac{t-t_0}{H}\right)^k \tag{2}$$

Substituting Eq. (1) into Eq. (2), we can obtain the Taylor series expansion of the $x(t)$ at t_0

$$x(t) = \sum_{k=0}^{\infty} \frac{(t=t_0)^k}{k!}\left[\frac{d^k c(t)}{dt^k}\right]_{t=t_0} \tag{3}$$

The function $x(t)$ is usually considered as a series with limited terms and Eq. (2) can be rewritten as:

$$x(t) \approx \sum_{k=0}^{m} X(k)\left(\frac{t-t_0}{H}\right)^k \tag{4}$$

where m represents the number of Taylor series' components. Usually, through elevating this value, the accuracy of the solution can be increased.

Some of the properties of DTM have been shown in Table 1. These properties are extracted from Eqs. (1) and (3).

3. Description of the problem

Consider a one-dimensional longitudinal fin with variable cross section A_c and arbitrary profile $F(X)$ (see Fig. 1). The length and perimeter of the fin are denoted by L and P, respectively. The fin extends to an ambient fluid with temperature T_a and is attached to a fixed prime surface with the temperature T_b. The fin thickness and base thickness are denoted by δ and δ_b, respectively. The fin is initially at ambient temperature. The temperature of the base fin is kept at the T_b. The energy balance equation of the fin, based on the one-dimensional heat conduction is given by:

$$\begin{aligned}\rho c \tfrac{\partial T}{\partial t} = \tfrac{\partial}{\partial X}\left(\tfrac{\delta_b}{2}F(X)K(T)\tfrac{\partial T}{\partial X}\right) - \tfrac{P}{A_c}H(T)(T-T_a) \\ +(A+B(T-T_a))\tfrac{\delta_b}{2}F(X)\end{aligned} \tag{5}$$

where ρ is density, c is specific heat capacity, X is space variable, t is time and T is temperature. In the above equation, the thermal conductivity K, convective heat transfer coefficient H and internal heat generation are assumed to be functions of temperature. A and B are the constant and temperature-dependent terms of the internal heat generation, respectively. Internal heat generation is per unit volume and so it has a direct relation with the fin profile. Previously, Partner and Raseelo [26] investigated this problem without considering internal heat generation.

The temperature of the base is suddenly changed to T_b and the fin tip is assumed adiabatic. Therefore, the thermal boundary conditions are in the following form [26]:

$$T(L,t) = T_b \ , \ \left.\frac{\partial T}{\partial X}\right|_{X=0} = 0. \tag{6}$$

The initial temperature of the fin is equal to the ambient temperature [26]:

$$T(X,0) = T_a \tag{7}$$

Table 1
The properties of the DTM.

Original function	DTM
$f(t) = g(t) \pm s(t)$	$F(k) = G(k) \pm S(k)$
$f(t) = cg(t)$	$F(k) = cG(k)$
$f(t) = \frac{d^n g(t)}{dt^n}$	$F(k) = \frac{(k+n)!}{H^n k!}G(k+n)$
$f(t) = g(t)s(t)$	$F(k) = \sum_{r=0}^{k} G(r)S(k-r)$